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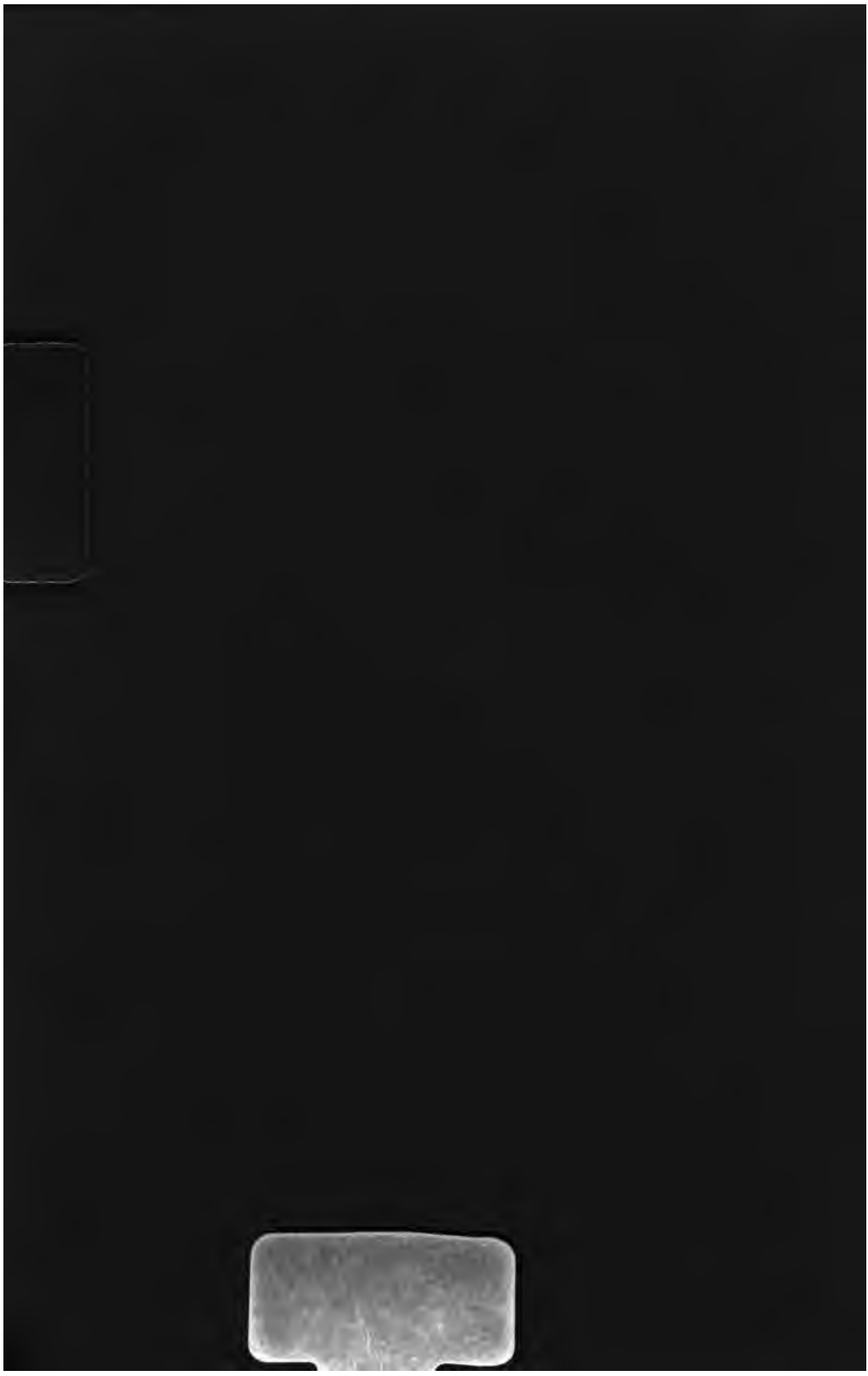

NAVIGATION

AND

NAUTICAL ASTRONOMY:

CONTAINING

PRACTICAL RULES, NOTES, EXAMPLES,

AND

AN APPENDIX,

GIVING EXPLANATIONS AND PROOFS OF THE RULES.

BY

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PREFACE.

THE following pages include, in a practical form, all the most useful Rules of Navigation and Nautical Astronomy.

The Rules have been made as clear and easy to understand as the nature of the subject would admit.

Notes are added to the Rules to render them applicable to particular cases; and numerous Examples with their Answers are given.

An extract from the Nautical Almanac is annexed, containing the Elements used in working the Examples.

January, 1869.

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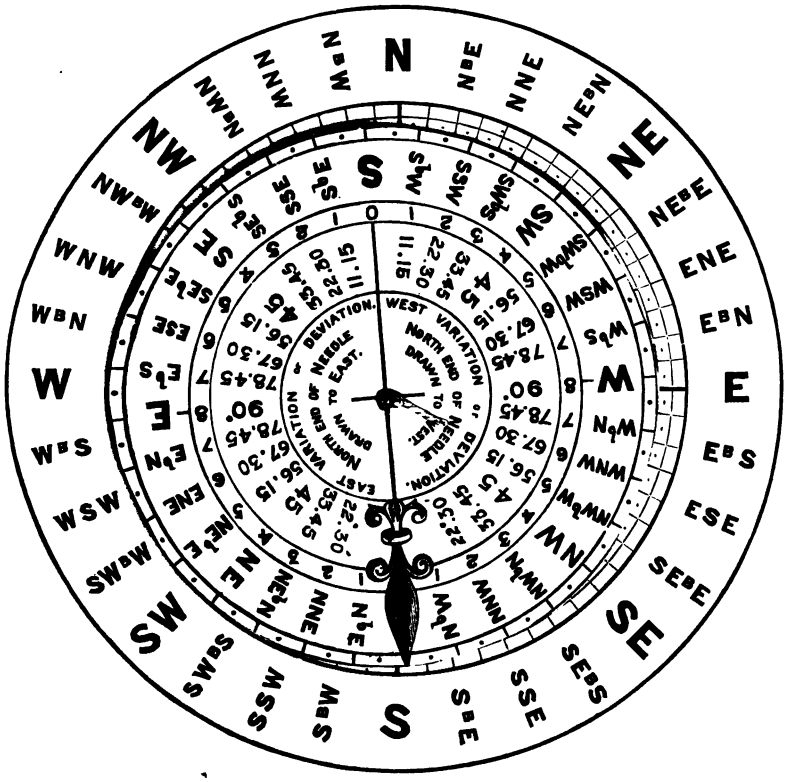
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NE
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WEST VARIATION
NORTH END OF NEEDLE DRAWN TO WEST.
DEVIATION.
NEEDLE EAST.
EAST VARIATION
NORTH END OF NEEDLE DRAWN TO EAST.

NAVIGATION.

NAVIGATION refers more especially to that branch of the art of conducting a ship from one part of the world to another which is performed by means of the mariner's compass and log line; and is usually divided into the following Rules, called the "Sailings," viz., Plane, Parallel, Traverse, Middle Latitude, and Mercator's Sailing, all of which are solved by the application of the rules of Plane Trigonometry.

PLANE SAILING.

IN this sailing, the distance being limited to so small a portion of the Globe as an ordinary day's run, no error of consequence can arise by considering

The meridians to be *straight* lines parallel to each other.

The parallels of latitude also to be *straight* lines cutting the meridians at right angles.

The rhumb lines* also *straight* lines.

The *figure* for plane sailing is a right-angled triangle, in which the vertical side (north and south line) represents the *difference of latitude.*

The horizontal side (east and west line) represents the *departure.*

The hypothenuse represents the *distance.*

The *angle* contained between the meridian and the distance represents the course.

Hence all plane sailing questions are merely solutions of *right-angled plane triangles.*

The two cases usually required in this sailing are:—

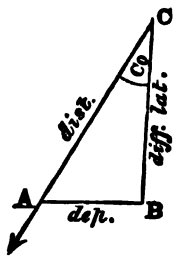
(1) When the course and distance are given, to find the difference of latitude and departure.

(2) When the difference of latitude and departure are given, to find the course and distance.

* The *rhumb* line is that line upon the surface of the terrestrial globe which makes the same angle with all the *meridians*, as it crosses them in succession.

PLANE SAILING.

Example 1.—A ship sails S. 30° W. until her distance is 120 nautical miles; find the difference of latitude and departure she has made.



Given { Co. S. 30° W. / dist. 120m. } to find { diff. lat. and dep.

To find the diff. lat.

$\frac{BC}{AC}$ or $\frac{\text{diff. lat.}}{\text{dist.}}$ = cos. course.

$\therefore$ diff. lat. = dist. cos. course (1)

$\therefore$ log. diff. lat. = log. dist. + Log. cos. co. − 10.

To find the dep.

$\frac{AB}{AC}$ or $\frac{\text{dep.}}{\text{dist.}}$ = sin. course.

$\therefore$ dep. = dist. sin. course. (2)

$\therefore$ log. dep. = log. dist. + Log. sin. co. − 10.

(1)		(2)	
log. dist. 120	2·079181	log. dist. 120	2·079181
Log. cos. co. 30°	9·937531	Log. sin. co. 30°	9·698970
log. diff. lat.	2·016712	log. dep.	1·778151

$\therefore$ diff. lat. = 103·9 miles. $\therefore$ dep. = 60 miles.

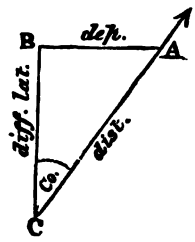
Note.—In every log. sin., log. cos., &c., the Tabular logarithm has 10 added to its *index*, and therefore 10 must be subtracted from the results above.

To find the diff. lat. and dep. by Traverse Table (by Inspection).

Enter the Traverse Table with the given distance 120, at the top (as dist.), and on the left hand side of the page look for 30°, as a course, and the corresponding numbers in the diff. lat. column will be 103·9, and in the dep. column will be 60, the diff. lat. and dep. required.

Example 2.—A ship makes 80 miles diff. lat. (northing), and 55 miles dep. (easting); find the course and distance.

Given { diff. lat. = 80 m. / dep. = 55 m. } to find { Course and dist.



To find the course.

$$\text{tan. co.} = \frac{AB}{BC} \text{ or } \frac{\text{dep.}}{\text{diff. lat.}} \quad (1)$$

$\therefore$ Log.tan.co. − 10 = log.dep. − log.diff.lat.

or, Log.tan. co. = 10 + log. dep. − log.diff.lat.

To find the dist.

$$\frac{AC}{BC} \text{ or } \frac{\text{dist.}}{\text{diff. lat.}} = \text{Sec. course.}$$

$\therefore$ dist. = diff. lat. sec. co. (2)

$\therefore$ log.dist. = log.diff.lat. + Log.sec.co. − 10.

(1)			(2)		
10 + log. dep.	55	11·740363	log. diff. lat.	80	1·903090
log. diff. lat.	80	1·903090	Log. sec. co.	34° 30′ 30″	10·084050
Log. tan. course		9.837273	log. dist.		1·987140

$\therefore$ course N. 34° 30′ 30″ E.*

$\therefore$ dist. 97 miles.

To find the course and distance by Traverse Table (by Inspection).

Seek in the Traverse Table for diff. lat. 80, and dep. 55, and find where these numbers, or the nearest numbers to those come together. In this instance we cannot find them to agree exactly, but have 80·4 in the diff. lat. column, and 54·2 in the dep. column, which give the course 34° (on the left hand side of the page), and the dist. 97 (at the top of the page). Hence the course is N. 34° E. (nearly), and the dist. 97m.

* To determine the *direction* of the course sailed. By the question the ship has made *northing* and *easting*, she has therefore gone *between* N. and E., having made an angle of 34° 30′ 30″ with the *meridian*, which angle we must indicate by the letters N. and E. to denote the quarter of the compass sailed in: hence the *course* is N. 34° 30′ 30″ E.

PLANE SAILING.

EXAMPLES FOR EXERCISE.

1. A ship sails from Lisbon N.W. by W., 128m.; required her diff. lat. and dep.—*See* Ex. 1, page 2.

Ans. Diff. lat. 71·1 miles, dep. 106·4 miles.

2. A ship from lat. 22° 55′ S. sails S.E. ½ E. 80 miles; find her lat. arrived at and dep. made.—*See* Ex. 1, page 2, and Rule (*a*) page 12.

Ans. Lat. arrived at 23° 45′ S., dep. 61·8 miles.

3. A ship from lat. 42° 20′ N. sails S. 80° E., for 20h. at the rate of 9 knots; find her lat. in and dep. made.

Ans. Lat. arrived at 41° 48′ N., dep. 177·3 miles.

4. A ship from lat. 1° 20′ N. sails S. 20° E., 200m.; required her lat. in and dep. made.

Ans. Lat. in 1° 48′ S., dep. 68·4 miles.

5. A ship sails N. 110m. and then E. 60m.; required her course and dist. made good.—*See* Ex. 2, page 3.

Ans. Course made good N. 28° 36′ 30″ E., dist. 125·3 miles.

6. A ship sails from lat. 20° 10′ N. until she arrives in lat. 23° 24′ N., having made 140m. of easting; required her course and distance made good.

Ans. Course made good N. 35° 49′ 0″ E. dist. 239·2.

7. A ship in lat. 48° 30′ S. sails on that parallel until she is 78m. to the westward of the place started from, and then steers S. until she arrives in lat. 50° 20′ S.; required her course and distance made good.

Ans. Course made good S. 35° 20′ 30″ W., dist. 134·8 miles.

8. A ship sails S. 50m. and then E. 50m.; required the course and distance made good.

Ans. Course S.E., or S. 45° E., dist. 70·7 miles.

9. A ship from San Francisco in lat. 37° 48′ N. sails in the N.W. quarter 115m., and then finds that she is 80m. west of the

meridian of that place; required the course steered, and latitude of the ship.

Ans. Course steered N. 44° 4′ 45″ W., lat of ship 39° 11′ N.

10. A ship from lat. 50° 35′ N. sails N. 20° W., until she has elevated the N. Pole 2°; required her lat. in and also the dist. and dep. made good.

Ans. Lat. in 52° 35′ N., dist. 127·7 miles, dep. 43·7 miles.

11. A ship from lat. 60° 40′ S. sails N. by E. until she has depressed the S. Pole 3°; required her lat. in, and distance run.

Ans. Lat. in 57° 40′ S., dist. 183·5 miles.

12. A ship from lat. 30° 20′ S. sails 100m. in the S.W. quarter, and finds that she is in lat. 31° 40′ S.; required the course and dep.

Ans. Course S. 36° 52′ 15″ W., dep. 60 miles.

13. Two ports are on the same parallel of lat., and 58m. distant. A ship sails on the meridian of the easterly one until she is 64m. north of it, and then speaks a barque that had sailed from the westerly port; required the course and distance of the barque.

Ans. Course N. 41° 11′ E., dist. 86·4 miles.

14. Two ships under the same meridian and 86 miles distant are to the eastward of a port, which is 50 miles from the northernmost ship, and on the same parallel with her; required the course and dist. of each ship to reach the port.

Ans. 1st course W. dist. 50m., 2nd course N. 30° 10′ 30″ W., dist. 99m.

15. A ship on the meridian of Fayal, in lat. 38° 32′ N., sails from lat. 35° 53′ N. on a direct course until she meets a brig which had run due W. 80m. from that place; required the course steered by the ship, and her distance run.

Ans. Course steered N. 26° 42′ 30″ W., dist. 178 miles.

PARALLEL SAILING.

When a ship sails on a parallel of latitude from a given meridian, *she changes her longitude only;* but the *distance between any two meridians* varies according to the parallel sailed on, and since longitude is measured on the equator, the spherical form of the Earth's surface must be taken into consideration before we can determine the relation between the departure, the difference of longitude, and the latitude of the parallel.

The method by which this relation is ascertained is called *parallel sailing.*

In this sailing a ship's track is supposed to be on a *parallel of latitude,* as from A to B. (*See* Fig. Ex. A.) The latitude of A or of B is represented by AD or BL.

The *diff. long.* between A and B, is the arc of the equator DL.

The arc AB, is the *departure* (or meridian distance) between A and B.

By Spherical Trigonometry.

It may be proved that,

$$\frac{\text{The arc AB of the parallel}}{\text{The arc DL of the equator}} = \text{cos. AD or cos. BL.}$$ (*See* Fig. Ex. A.)

that is, $\frac{\text{dep.}}{\text{diff. long.}} = \text{cos. lat.}$; whence the following formulæ.

(A) $\frac{\text{dep.}}{\text{diff. long.}} = \text{cos. lat.}$

$\therefore$ dep. = diff. long. cos. lat.

When the *lat.* of the parallel and diff. long. between the two places are given; to find the *dep.* (or mer. dist.) between the two places situated on that parallel.

(B) $\frac{\text{diff. long.}}{\text{dep.}} = \text{sec. lat.}$

$\therefore$ diff. long. = dep. sec. lat.

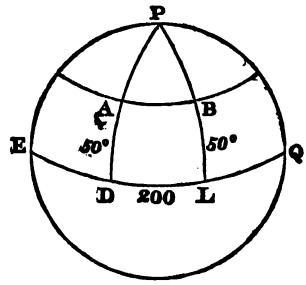
When the *dep.* (or mer. dist.) between the two places, and the lat. of the parallel are given; to find the diff. long. between the two places.

(C) $\text{cos. lat.} = \frac{\text{dep.}}{\text{diff. long.}}$

When the *dep.* (or mer. dist.) and *diff. long.* between the two places are given; to find the lat. of the parallel on which the two places are situated.

EXAMPLE (A).

A ship on the parallel of 50° N. (or S.) latitude, makes 200 miles difference of longitude; find the corresponding departure (or meridian distance).



Given AD = BL = lat. 50°.
DL = diff. long. = 200 m.

To find AB = required departure (or meridian distance).

Then $\frac{AB}{DL}$ = cos. AD.

$\therefore \frac{\text{dep.}}{\text{diff. long.}}$ = cos. lat.

$\therefore$ dep. = diff. long. × cos. lat.

$\therefore$ log. dep. = log. diff. long. + Log. cos. lat. − 10.

log. diff. long. 200	2·301030
Log. cos. lat. 50°	9·808067
log. dep.	2·109097

$\therefore$ dep. (or mer. dist.) = 128·5 miles.

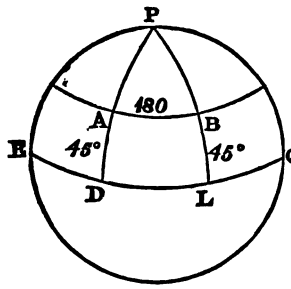
By Inspection.

With the latitude of the parallel as a *course*, and the diff. long. as a *distance*, enter the Traverse Table, and the corresponding *diff. lat.* will be the departure required.

Example.—Enter the Traverse Table with 50° as a *course*, and 200 in the *dist.* column: the corresponding *diff lat.* 128·6, will be the departure required.

EXAMPLE (B).

A ship on the parallel of 45° N. latitude, makes 180 miles of departure: find the corresponding difference of longitude.



Given AB = dep. (or mer. dist.) = 180m.
AD = BL = lat. 45°.

To find DL = required diff. long.

Then $\frac{DL}{AB}$ = sec. AD;

or, $\frac{\text{diff. long.}}{\text{dep.}}$ = sec. lat.

∴ diff. long. = dep. × sec. lat.

∴ log. diff. long. = log. dep. + Log. sec. lat. − 10.

log. dep. 180	2·255273
Log. sec. lat. 45°	10·150515
log. diff. long.	2·405788

∴ diff. long. = 254·5 miles.

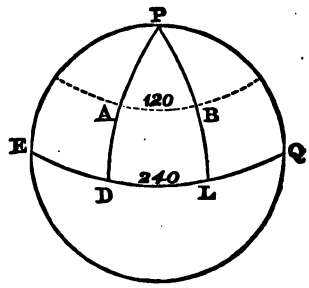
By Inspection.

With the latitude of the parallel as a *course*, and the dep. as *diff. lat.*, enter the Traverse Table, and the corresponding *dist.* will be the diff. long. required.

Example.—Enter the Traverse Table with 45° as *course*, and 180 (or the nearest number to it) as *diff. lat.*, the corresponding *distance*, 255, will be the diff. long. required.

EXAMPLE (C).

On what parallel of latitude must a ship sail 120 miles due E., or due W., to change her longitude 240 miles?



Given AB = dep. (or mer. dist.) = 120 m.
DL = diff. long. = 240 m.

To find AD = BL = latitude required.

Then cos. lat. $= \frac{\text{dep.}}{\text{diff. long.}}$

$\therefore$ Log. cos. lat. $-$ 10 = log. dep. $-$ log. diff. long.

or, Log. cos. lat. = 10 + log. dep. $-$ log. diff. long.

10 + log. dep.	120	12·079181
log. diff. long.	240	2·380211
Log. cos. lat.		9·698970

$\therefore$ Latitude of parallel = 60°.

By Inspection.

With the diff. long. as *distance*, and the dep. as *diff. lat.*, enter the Traverse Table, and the corresponding *course* will be the latitude required.

Example. Enter the Traverse Table with 240 as *dist.* and 120 (or the nearest number to it) as *diff. lat.*, and the corresponding *course*, 60°, will be the latitude required.

Note.—We can also solve these problems by constructing a right-angled plane triangle, in which the base represents the departure, the angle at the base the *latitude* of the parallel, and the hypothenuse represents the *difference of longitude.*

PARALLEL SAILING.

EXAMPLES FOR EXERCISE.

1. A ship on the parallel of 55° N. sails due E. from long. 35° 30′ W. to long. 25° 30′ W.; find the meridian distance (dep.) she has made.—*See* Example (A) page 7.

Ans. Dep., 344·1m.

2. A ship is on the parallel of Cape Race, in lat. 46° 40′ N., and long. 47° 8′ W.; how far must she sail in nautical miles to reach the Cape?

Ans. Dist. to sail, 246·3m.

3. Two ships in 60° S. are 200 nautical miles apart; what is their difference of longitude?—*See* Example (B) page 8.

Ans. Diff. long., 400m., or 6° 40′.

4. How far must a ship sail to reach the meridian of Valparaiso, in long. 71° 41′ W., if she is 10 miles *due* south of Cape Horn, in lat. 55° 59′ S., and long. 67° 16′ W.?

Ans. Dist. to sail, 147·9m.

5. The diff. long. between two ships is 500m., and their meridian distance is 250m.; what parallel are they on?—*See* Example (C) page 9.

Ans. The parallel of 60°.

6. A ship on the meridian of St. Paul's, in long. 77° 35′ E., and in lat. 54° S., sails due W. until she is on the meridian of Port Louis (Isle of France), in long. 57° 32′ E.; how many nautical miles must she have run?

Ans. Dist. run, 707·1m.

7. In what latitude will 300 miles of dep. equal 600 miles of diff. long.?

Ans. In lat. 60°.

8. How far must a ship sail due W. from Ascension (Atlantic), in lat. 7° 55′ S., and long. 14° 25′ W., to change her long. 3°?

Ans. Dist. 178·3m.

9. Being 150m. (in diff. long.) east of Halifax, in lat. 44° 40′ N., how far must I sail to reach the harbour?

Ans. Dist. to sail, 106·7m.

10. In what latitude must a ship be that each mile of dep. (mer. dist.) shall be equal to two miles of diff. long.?

Ans. In lat. 60° north or south.

11. How far must a ship sail due W. from Cape Clear, in lat. 51° 26′ N., and long. 9° 22′ W., to reach the meridian of Funchal (Madeira), in long. 16° 55′ W.?

Ans. Dist. to sail, 280m.

12. What is the length of a degree (in nautical miles) on the parallel of 70°?

Ans. Length of a degree, 20·52m.

13. How much longitude will a ship run down in a day, when in the lat. of Cape Horn, 55° 59′ S., if she sails due W. at the rate of 10 knots per hour?

Ans. 7° 9′.

14. Two vessels leave Fayal, in lat. 38° 32′ N., and long. 28° 38′ W., bound to Halifax, in lat. 44° 40′ N., and long. 63° 35′ W., one sails *due* N. until she is in the lat. of Halifax, and then steers *due* W. until she reaches it; the other sails *due* W. until she is in the long. of Halifax, and then steers *due* N. until she reaches it; which will arrive there first if they both sail at the same rate, and how many miles will she gain on the other?

Ans. The former vessel, and will gain 149 miles.

To find the latitude arrived at (lat. in) when the latitude left and diff. lat. are given.

RULES.*

(*a*) When the lat. left and diff. lat. are of the *same* name, their *sum* will be the lat. arrived at.

(*b*) When the lat. left and diff. lat. are of *contrary* names, take their difference, and the remainder will be the lat. arrived at, and of the name of the greater.

EXAMPLES.

1. A ship from a place A, in lat. 17° 40′ N. makes 130 miles diff. lat. (northing); find the latitude arrived at.

lat. left	17° 40′ N.
diff. lat. 130′ =	2 10 N.
lat. arrived at, or lat. in	19 50 N.

2. A ship leaves a place A, on the equator, and makes 114 miles diff. lat. (southing); what is her latitude in?

lat. left.	0° 0′
diff. lat. 114′ =	1 54 S.
lat. in (arrived at)	1 54 S.

3. A ship from a place A, in lat. 21° 15′ S., makes 160 miles diff. lat. (north); what is her lat. arrived at?

lat. from	21° 15′ S.
diff. lat. 160′ =	2 40 N.
lat. arrived at	18 35 S.

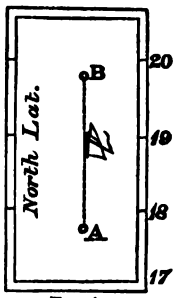
4. A ship from a place A, in lat. 1° 44′ N., makes 235 miles diff. lat. (south); what is her lat. arrived at?

lat. left	1° 44′ N.
diff. lat. 235′ =	3 55 S.
lat. arrived at	2 11 S.

* In order to remember these Rules, observe that, when a ship is receding from the equator, she is getting to a greater latitude; and when approaching it, to a less. In the former case, we *add* the diff. lat.; in the latter, *subtract* it.

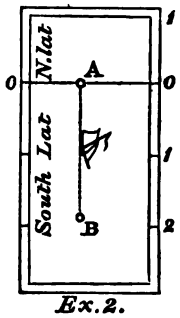
NOTES.

Example 1.—In this example the ship has gone 2° 10′ to the north, and consequently *increased* her distance from the equator.



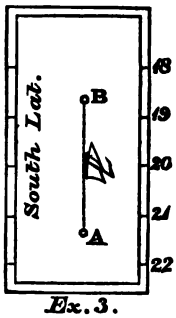
Ex. 1.

Example 2.—In this example the ship has made 1° 54′ diff. lat. to the south, the whole of which was from the equator itself; and therefore her diff. lat. is the same as the lat. arrived at.



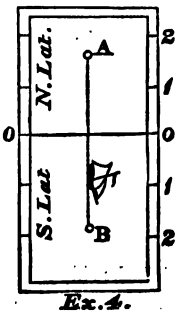
Ex. 2.

Example 3.—In this example the ship was in south lat., and has made 2° 40′ diff. lat. to the *north*, and consequently neared the equator by that number of miles, and *diminished* her south latitude.



Ex. 3.

Example 4.—In this example the diff. lat., 3° 55′ south, is *greater* than the lat. left, which was *north;* she has, therefore, *crossed* the equator and changed the name of her latitude to south.



Ex. 4.

To find the longitude arrived at (long. in) when the longitude left and diff. long. are given.

RULES.*

(*a*) When the long. left and diff. long. are of the *same* name, their sum will be the long. in arrived at.

(*b*) When the long. left and diff. long. are of *contrary* names, the difference between the long. left and diff. long. will be the long. arrived at, and of the name of the greater.

EXAMPLES.

1. A ship from a place A, in longitude 27° 50′ W., makes 50 miles diff. long. west; required the longitude arrived at.

long. left	27°	50′	W.
diff. long. 50′ =	0	50	W.
long. arrived at	28	40	W.

2. A ship from a place A, on the meridian of Greenwich, makes 127 miles diff. long. E.; required her long. in.

long. left	0°	0′	
diff. long. 127′ =	2	7	E.
long. in	2	7	E.

3. A ship leaves a port A, in longitude 178° 26′ E., and makes 264 miles diff. long. E.; what is her longitude arrived at?

long. left	178°	26′	E.
diff. long. 264′ =	4	24	E.
	182	50	E.
	360		
long. in (arrived at)	177	10	W.

In this case the sum of the long. left and diff. long. exceeds 180°, subtract from 360°, and change the name from E. to W.

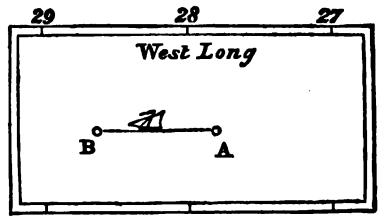
4. A ship from a place A, in longitude 2° 30′ E., makes 330 miles diff. long. W.; find her longitude arrived at.

long. left	2°	30′	E.
diff. long. 330′ =	5	30	W.
long. in (arrived at)	3	0	W.

* If the ship is receding from the first meridian, her longitude is getting *greater*; if approaching it, *less*.

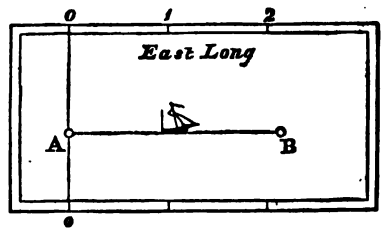
NOTES.

Example 1.—In this example the ship has made 50 m. diff. long. *west*, and consequently *increased* her *west longitude* by 50 m.



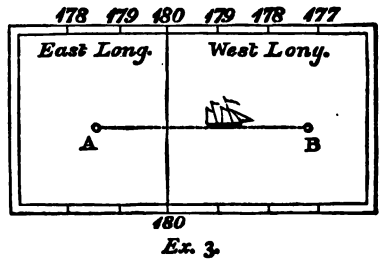
Ex. 1.

Example 2.—In this example the ship left the meridian of Greenwich, and has since made 2° 7′ diff. long. away from it to the east; hence her long. arrived at is equal to her diff. long.



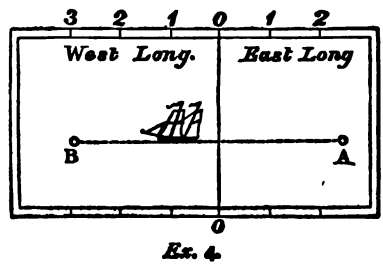
Ex. 2.

Example 3.—In this example the ship has gone from east into west longitude, and by crossing the meridian of 180°, has changed the *name* of her longitude from east to west.



Ex. 3.

Example 4.—In this example the ship has made 5° 30′ diff. long. west, and thereby crossed the meridian of Greenwich, having run from east into west longitude.



Ex. 4.

TRAVERSE SAILING.

IN the navigation of a ship it frequently happens that, from the prevalence of contrary winds and other causes, she cannot be kept on the *same course* for any number of hours, and thus, during a day, or any given period of time, she will sail on several courses. In order to reduce these to a *single* course and distance, as well as to determine the difference of latitude and departure she has made, we have the assistance of a Table so constructed as to give the *diff. lat.* and *dep.* for every *course* from $\frac{1}{4}$ to $7\frac{3}{4}$ points, as well as from 1° to 89° and every *distance* from 1 to 296 miles; (*see* Inman, Table X.),* the use of which is given in the following page.

Draw out a Form (called a Traverse Table), as under.

Corrected Courses.	Dist.	DIFF. LAT.		DEP.	
		N.	S.	E.	W.
(1)	(2)	(3)	(4)	(5)	(6)

In the first column (1) write down the several corrected courses.

In the second column (2) write down the distances made on the several courses.

In the third and fourth columns, (3) and (4), are to be placed the differences of latitude taken from Table X.

In the fifth and sixth columns, (5) and (6), are to be placed the departures, taken from Table X.

* *The lower division of the Table gives the Course in* Degrees.

Before commencing to take out the diff. lat. and dep. corresponding to the several courses and distances, draw a line ——— through each column that will not be required.

Note 1.—When the distance exceeds 296m. (the limits of the Table) divide it by 2, and take out the diff. lat. and dep. for this new distance, *multiply* them by 2, and place the results in their proper columns as in ordinary cases.

Note 2.—When there is a decimal place in the distance look for it as a whole number in the Traverse Table (X.), and point off one place to the left in the diff. lat. and dep. found, and in this form set them down in the Traverse Table.

To find the DIFF. LAT. *and* DEP. *corresponding to a given course and distance.*

When the Course is less than 4 Points.

Enter Table X. with the given course on the *left-hand* side of the page, and look for the distance at the top of the page; take out the corresponding *diff. lat.* and *dep.*, placing the diff. lat. in the N. or S. column, (3) or (4) of the foregoing form, according as the course contains *northing* or *southing;* and the *dep.* in the E. or W. column, (5) or (6), according as the course contains *easting* or *westing.*

When the Course is *more* than 4 Points.

Take the course from the *right-hand* side of the page, in which case the columns of diff. lat. and dep. become reversed, that is, diff. lat. at the *top* of the page, is dep. at the *bottom* of the page; and *dep.* at the top, is *diff. lat.* at the bottom of the page.

When a Course is *due* N. or *due* S.

The distance run will be the *diff. lat.*, and must be placed in the N. or S. column, according as the ship has gone due N. or due S.

When a Course is *due* E. or *due* W.

The distance will be the *dep.*, and must be placed in the E. or W. column, according as the ship has gone due E. or due W.

Course given in *Degrees.*

It must be looked out in the *lower* division of Table X. When not more than 45°, on the left-hand side of the page; and when it exceeds 45°, on the right-hand side of the page.

To find the DIFF. LAT. *and* DEP. *made good.*

After the N. S. E. and W. columns have been filled in with the differences of lat. and departure corresponding to the several courses and their distances, add up each column separately. Then take the less diff. lat. from the greater and mark the remainder N. or S., according to which is the greater.

Also take the less dep. from the greater, and mark the result E. or W., according as the easting or westing is the greater.

These remainders will be called the diff. lat. and dep. *made good* for the day, or any other given interval.

EXAMPLE, to illustrate the foregoing Rules.

A ship from latitude 50° 9′ N., and longitude 12° 35′ W., sails S.S.W. 25 miles, N.W.½N. 30 miles, W. by S. 18 miles, E. 10 miles, and S. 15 miles; find the diff. lat. and dep. she has made.

POINTS from N. or S.	COURSES.	Dist.	DIFF. LAT.		DEP.	
			N.	S.	E.	W.
2	S.S.W.	25	—	23·1	—	09·6
3½	N.W.½N.	30	23·2	—	—	19·0
7	W. b. S.	18	—	03·7	—	18·6
8	E.	10	—	—	10·0	—
0	S.	15	—	15·0	—	—
			23·2	41·8	10·0	47·2
				23·2		10·0
				18·6*		37·2†
				* Diff. Lat. made good S.		† Dep. made good W.

Working of the above Example.

Find the number of *points* (reckoned *from* N. or S.) corresponding to each course; then enter Table X. and take out the differences of lat. and departures, as follows:—

1st Course. Enter the Table with (S.S.W.) 2 points, on the *left-hand* side of the page, and 25m. at the top. This will give 23·1 *diff. lat.* to be placed in the S. column (because the ship has made southing), and 09·6 dep. to be placed in the W. column (because the ship has made westing).

2nd Course. Enter the Table with (N.W.½N.) 3½ points, and dist. 30m., which gives 23·2 for the N. column, and 19·0 for the W. column.

3rd Course. Enter the Table with (W. by S.) 7 points on the *right-hand* side of the page, and distance 18m. at the *top*. This gives 03·7 for the diff. lat., to be placed in the S. column, and 18·6 dep. to be placed in the W. column. (*See* remark, when course is *greater* than 4 points.)

4th Course. The Table is not required in this case, the ship having made all easting, the dist. 10m. must be placed as a *dep.* in the E. column.

5th Course. The Table is not required in this case, the ship having made all southing, the dist. 15m. is the *diff. lat.*, and must be placed in the S. column.

Now add up the N. S. E. and W. columns separately; take the N. diff. lat. from the S. diff. lat., and the remainder 18·6 will be the diff. lat. *made good*, and must be marked S. because the ship has made most *southing*.

Also, take the E. dep. from the W. dep., the remainder 37·2 will be the dep. *made good*, and is W. because it was greater than the *easting*.

Hence, the result of all the separate courses and distances gives 18·6 diff. lat. S., and 37·2 dep. W.

TRAVERSE SAILING.

EXAMPLES FOR EXERCISE.

1. A ship sails N. by E. 30 miles, S.S.W. 15 miles, S.E. by S. 20 miles, N.N.W. 16 miles, S. 14 miles, and N.N.E. 18 miles; find the diff. lat. and dep. made good.—*See* Example, page 18.

Ans. Diff. lat. 16·3 N. Dep. 12·1 E.

2. A ship sails S.S.E.$\frac{1}{2}$E. 14 miles, S.W.$\frac{1}{2}$S. 19 miles, N.E. by N.$\frac{1}{2}$N. 21 miles, N.W. 14 miles, S.S.E.$\frac{1}{2}$S. 27 miles, and S.$\frac{1}{2}$W. 24 miles; find the diff. lat. and dep. made good.

Ans. Diff. lat. 48·3 S. Dep. 0·0.

3. A ship sails S.W. by S. 16 miles, S.W. by W. 20 miles, N.E.$\frac{1}{2}$E. 17 miles, E.$\frac{1}{2}$S. 10 miles, W.N.W.$\frac{1}{2}$W. 13 miles, and S.S.W.$\frac{1}{4}$W. 22 miles; find her diff. lat. and dep., and also her course and distance made good.

Ans. Diff. lat. 30·7 S. Course S. 38° 14′ 45″ W. Dep. 24·2 W. Dist. 39m.

4. A ship from lat. 16° 10′ N., and long. 16° 10′ W., sails N.W. by W. 27 miles, N.N.E.$\frac{3}{4}$E. 60 miles, E.$\frac{1}{2}$S. 42 miles, and S.S.W.$\frac{1}{2}$W. 57 miles; find her course and distance made good, and also her lat. and long. arrived at.

Ans. Course N. 63° 32′ 30″ E. Lat. in 16° 22′ N. Dist. 26·26m. Long. in 15° 45′ 42″ W.

5. A ship from Fernando de Noronha, in lat. 3° 52′ S., sails N. 28° W., 39 miles, N. 45° W. 16 miles, S. 78° W. 27 miles, S. 62° W. 31 miles; find her course and distance made good, and also the lat. and long. she has reached.

Ans. Course N. 73° 0′ W. Lat. in 3° 26′ 30″ S. Dist. 87·2m. Long. in 33° 51′ W.

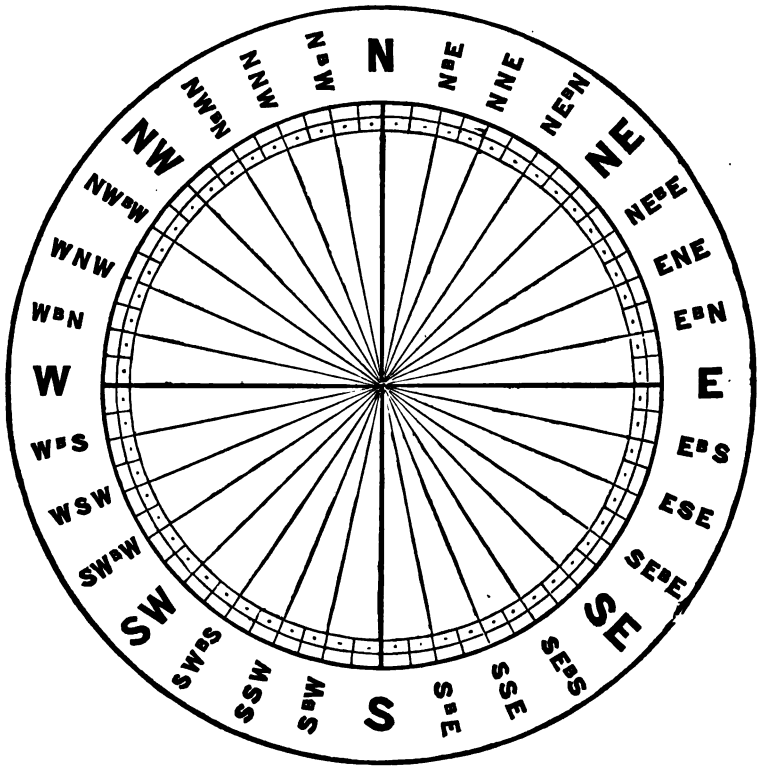
6. A ship on the Equator, and in long. 20° 16′ W., sails N.N.E.$\frac{1}{2}$E. 14 miles, W.S.W. 21 miles, E.$\frac{3}{4}$S. 30 miles, S.S.E.$\frac{1}{4}$E. 21 miles, and N.N.W.$\frac{1}{4}$W. 15 miles; find her course and distance made good, and also the lat. and long. in.

Ans. Course S. 74° 15′ E. Lat. in 0° 5′ 30″ S. Dist. 20·26m. Long. in 19° 56′ 30″ E.

7. A ship from lat. 54° 15′ S., and on the meridian of Greenwich, sails S. 54° W. 32 miles, S. 71° E. 40 miles, S. 10° W. 44 miles, N. 78° W. 15 miles, and N. 45° E. 27 miles; find her lat. and long. in.

Ans. Lat. in 55° 8′ S. Long. in 0° 15′ E.

THE COMPASS.



Points.	From N.		From S.		Degrees.		
	N.	N.	S.	S.	°	′	″
¼	N. ¼ E.	N. ¼ W.	S. ¼ E.	S. ¼ W.	2	48	45
½	N. ½ E.	N. ½ W.	S. ½ E.	S. ½ W.	5	37	30
¾	N. ¾ E.	N. ¾ W.	S. ¾ E.	S. ¾ W.	8	26	15
1	N. b E.	N. b W.	S. b E.	S. b W.	11	15	0
2	N. N. E.	N. N. W.	S.S.E.	S.S.W.	22	30	0
3	N.E. b N.	N.W. b N.	S.E. b S.	S.W. b S.	33	45	0
4	N.E.	N.W.	S.E.	S.W.	45	0	0
5	N.E. b E.	N.W. b W.	S.E. b E.	S.W. b W.	56	15	0
6	E.N.E.	W.N.W.	E.S.E.	W.S.W.	67	30	0
7	E. b N.	W. b N.	E. b S.	W. b S.	78	45	0
8	East	West	East	West	90	0	0

TABLE *

For converting Points of the Compass and their fractional parts into Degrees.

Points.	No.	Degrees.	Points.	No.	Degrees.
		° ′ ″			° ′ ″
NORTH.	0	0 0 0	N.E.	4	45 0 0
	1/8	1 24 22		1/8	46 24 22
1/4 —		2 48 45	1/4 —		47 48 45
	3/8	4 13 7		3/8	49 13 7
1/2 —		5 37 30	1/2 —		50 37 30
	5/8	7 1 52		5/8	52 1 52
3/4 —		8 26 15	3/4 —		53 26 15
	7/8	9 50 37		7/8	54 50 37
N. b E.	1	11 15 0	N.E. b E.	5	56 15 0
	1/8	12 39 22		1/8	57 39 22
1/4 —		14 3 45	1/4 —		59 3 45
	3/8	15 28 7		3/8	60 28 7
1/2 —		16 52 30	1/2 —		61 52 30
	5/8	18 16 52		5/8	63 16 52
3/4 —		19 41 15	3/4 —		64 41 15
	7/8	21 5 37		7/8	66 5 37
N.N.E.	2	22 30 0	E.N.E.	6	67 30 0
	1/8	23 54 22		1/8	68 54 22
1/4 —		25 18 45	1/4 —		70 18 45
	3/8	26 43 7		3/8	71 43 7
1/2 —		28 7 30	1/2 —		73 7 30
	5/8	29 31 52		5/8	74 31 52
3/4 —		30 56 15	3/4 —		75 56 15
	7/8	32 20 37		7/8	77 20 37
N.E. b N.	3	33 45 0	E. b N.	7	78 45 0
	1/8	35 9 22		1/8	80 9 22
1/4 —		36 33 45	1/4 —		81 33 45
	3/8	37 58 7		3/8	82 58 7
1/2 —		39 22 30	1/2 —		84 22 30
	5/8	40 46 52		5/8	85 46 52
3/4 —		42 11 15	3/4 —		87 11 15
	7/8	43 35 37		7/8	88 35 37
N.E.	4	45 0 0	EAST.	8	90 0 0

* The remaining Quarters of the Compass may be converted into Points or Degrees by aid of this Table.

VARIATION OF THE COMPASS.

CORRECTING COURSES.

THE direction of the North point of the magnetic (or compass) needle, is not the same in all places, but is drawn aside more or less to the *right* or *left* of the true north point of the world.

The angle included between the direction of the Magnetic and true N. points, is called the VARIATION of the Compass.

[A] When the compass N. is drawn to the *right* of the true N. the variation is easterly.

[B] When the compass N. is drawn to the *left* of the true N. the variation is westerly.

In consequence of this divergence of the magnetic needle from the *true* meridian, the courses steered or bearings observed by compass will not correspond to the courses or bearings given by a chart, since charts are so constructed that the lines upon them shall represent the *true* points of the world, and therefore, whenever it is required to turn a course or bearing by compass into a course or bearing by chart, the variation of the compass must be applied.

The manner in which this is done may be shown as follows :—

Let two compass cards be placed one over the other, so as to have the same centre. The lower one to be *fixed*, and represent the true points of the world, whilst the upper one is *moveable*, and represents the magnetic compass; then, if the two N. points are made exactly to coincide, there would be *no* variation.

Again, suppose the *magnetic* N. to be moved aside 3 points to the *right* of the true N., then the N.W. by N. point of the compass would agree with the N. point of the world, and there would be 3 points easterly variation of the compass. In other words, when the ship is *steering* N.W. by N. by compass, she is in reality making a N. course by the world; and in order

to *correct* the compass course N.W. by N., so as to get N. (true), the variation must be allowed to the *right.*

[C] Hence, to correct a compass course for *easterly* variation, it must be allowed to the *right,* to obtain the *corresponding* true course.

Again, suppose the *magnetic* N. to be moved 4 points to the *left* of the true N., then the N.E. point of the compass would agree with the N. point of the world, and there would be 4 points westerly variation of the compass. In other words, when the ship is steering N.E. by compass, she is in reality making a N. course by the world; and in order to *correct* the compass course N.E., so as to get N. (true), the variation must be allowed to the *left.*

[D] Hence, to correct a compass course for *westerly* variation, it must be allowed to the *left,* to obtain the *corresponding* true course.

The **true** course denotes the course as given by the *chart,** unaffected either by variation or by deviation; and is the course that would be steered by the compass if there were neither variation nor deviation.

The **correct magnetic** course denotes the course as given by the compass, affected by variation *only;* and is the course that would be steered by the compass if there were no deviation.

The **compass** course is influenced both by variation and by deviation.

* A few of the Admiralty Charts are drawn for convenience on the *magnetic* meridian; the navigator would of course pay attention to this occasional departure from the general practice of all charts and plans in this country being drawn on the *true* meridian.

CORRECTING MAGNETIC COURSES OR BEARINGS FOR VARIATION.

When the variation is easterly, allow it to the *right* of the correct magnetic course, to obtain the true course.

When the variation is westerly, allow it to the *left* of the correct magnetic course, to obtain the true course.

Note.—When allowing the variation, consider yourself at the CENTRE of the *compass*, and facing the point you wish to correct.

EXAMPLES.

Magnetic Courses.	Variation.	True Courses.	Magnetic Courses.	Variation.	True Courses.
N.N.W.	1 E.	N. b W.	N.N.E.	1 W.	N. b E.
N.W. b N.	1 „	N.N.W.	N.E. b N.	1 „	N.N.E.
N.W. b W.	2 „	N.W. b N.	N.E. b E.	2 „	N.E. b N.
W.S.W.	2 „	West	E.N.E.	2 „	N.E.
S.W.	4 „	West	N.E.	4 „	North
S. b W.	2 „	S.W. b S.	N. b E.	2 „	N. b W.
S. b E.	1 „	South	E. b S.	1 „	East

CORRECTING TRUE COURSES OR BEARINGS FOR VARIATION.

Correcting a *true* course for variation being exactly the *reverse operation* of correcting a correct magnetic course for variation, we have the following rules:—

When the variation is easterly, allow it to the *left* of the true course, to obtain the correct magnetic course.

When the variation is westerly, allow it to the *right* of the true course, to obtain the correct magnetic course.

EXAMPLES.

True Courses.	Variation.	Magnetic Courses.	True Courses.	Variation.	Magnetic Courses.
N.N.E.	2 E.	North	N.N.W.	2 W.	North
N.E. b N.	2 „	N. b E.	N. b W.	2 „	N. b E.
N.E.	1 „	N.E. b N.	N.W. b. N.	1 „	N.N.W.
E.N.E.	2 „	N.E.	W.N.W.	1 „	N.W. b W.
East	2 „	E.N.E.	West	2 „	W.N.W.
E. b S.	3 „	E.N.E.	W.S.W.	2 „	West
S.E. b E.	3 „	East	S.W.	1 „	S.W. b W.
S.E. b S.	1 „	S.E.	S.S.W.	1 „	S.W. b S.
S.S.E.	2 „	S.E.	South	2 „	S.S.W.

CORRECTING STANDARD COMPASS COURSES OR BEARINGS FOR LOCAL DEVIATION.

The variation of the compass, before noticed, is caused by the magnetism of the *Earth* acting upon the needle, and deflecting it from the *true* meridian of the world; but when a compass is used on board a ship, it is subject also to another disturbance, generally known to seamen as *local deviation.*

This additional source of error in the *direction* of the needle arises from the magnetism of the various masses of iron surrounding it, such as armour-plating, guns, tanks, engines, &c.

In every ship the compass has a *local deviation*, but in some ships it is much greater than in others.

The amount of *local deviation* is contained in the angle between the *correct magnetic* direction of the ship's head and the direction of the ship's head as shown by the *affected* compass.

The deviation is named *easterly*, when the north end of the needle is drawn to the *right* of the magnetic meridian by the iron of the ship.

The deviation is named *westerly*, when the north end of the needle is drawn to the *left* of the magnetic meridian by the iron of the ship.

Since the deflection of the compass needle from the magnetic meridian (*local deviation*) is similar in effect to that of the deflection of the magnetic from the true meridian (*variation*), the local deviation must be applied to the ship's courses, or compass bearings, in the same manner as the variation, viz.:—

(*a*) When the deviation is *easterly*, it must be allowed to the *right* of the compass course to obtain the *correct magnetic course.*

(*b*) When the deviation is *westerly*, it must be allowed to the *left* of the compass course to obtain the *correct magnetic course.*

Note.—In iron ships an error is caused in the deviation when the ship "heels" to starboard or port. In some ships the error, when the ship's head is N. or S., is equal to, or even greater than the amount of heel. In other words, a "heel" of 10° may give rise to a deviation of 10° or more.

LEEWAY.

WHEN a ship is under sail, and the wind is blowing on her side, she is liable to be pressed by its force to *leeward* of the course she is endeavouring to steer. This lateral or side-way pressure is the cause of the *Leeway*, and its amount can be ascertained by observing the angle made by the direction of her keel and the line of wake she leaves on the surface of the water.

Since the effect of the wind is to drive the ship to the *left* of her intended course when she is on the *Starboard* tack; and to the *right* of her intended course when she is on the *Port* tack, we have the following rules for allowing the *Leeway*:—

(*a*) When the ship is on the Starboard tack, the leeway must be allowed to the *left* of the course steered.

(*b*) When the ship is on the Port tack, the leeway must be allowed to the *right* of the course steered.

CORRECTING COURSES FOR LEEWAY.

EXAMPLES.

Courses.	Winds.	Leeway.	Courses made good.	Courses.	Winds.	Leeway.	Courses made good.
	STAR. TACK.				PORT TACK.		
N.N.E.	East	2	North	W.S.W.	South	1	W. b S.
N.W. b N.	N.E b N.	1	N.W.	N.W. b W.	S.W. b W.	3	N.N.W.
E.N.E.	S.E.	3	N.E. b N.	N.E.	N.N.W.	2	E.N.E.
S.W. b. W.	N.W. b W.	2	S.W. b S.	E.N.E.	N.N.E.	1	E. b N.
S.S.E.	S.W.	3	S.E. b E.	E.S.E.	N.E.	2	S.E.
S. b W.	W. b S.	2	S. b E.	S.S.E.	East	3	S. b W.
S.E. b E.	S. b W.	1	E.S.E.	South	E.S.E.	3	S.W. b S.
East	S.S.E.	3	N.E. b E.	S.S.W.	S.E.	2	S.W.
N.E. b E.	S.E. b E.	2	N.E. b N.	W.S.W.	South	1	W. b S.
N.E.	E.S.E.	1	N.E. b N.	W. b N.	S.W. b S.	2	N.W. b W.

MIDDLE LATITUDE SAILING.

THIS sailing is founded on a supposition that the meridian distance *half-way* between the latitude *sailed from*, and that *bound to*, is equal to the departure which a ship makes in sailing from the one latitude to the other; and is a compound of *Plane* and *Parallel* sailing.

The two cases most commonly used in this sailing are—

(1) When the *latitudes* and *longitudes* of the two places are given } To find the course and distance between them.

(2) When the *course* between the two places and their *latitudes* are given } To find the difference of longitude.

CASE (1).

Find the true course and distance from one place to another.

(*a*) On the left hand of the page, write down the latitude of the place from, underneath which put the latitude of the place bound to, or worked for.

(*b*) In the middle of the page, write down the latitude of the place from, underneath which put the latitude of the place bound to, or worked for.

(*c*) On the right hand of the page, write down the longitude of the place from, underneath which put down the longitude of the place bound to, or worked for.

(A) When the latitudes are both of the *same* name, take the less from the greater; but when they are of contrary names, take their *sum*; the result, in either case, multiplied by 60 will give the difference of latitude, *in miles*.

(B) Find the middle latitude as follows:—

(1) When the latitudes are both of the *same* name, half their SUM will give the *middle* latitude.

(2) When the latitudes are of *contrary* names, half the GREATER latitude will give the middle latitude.

(C) When the longitudes of the two places are both of the same name, take the less from the greater; but when they are of contrary names, take their sum; the result, in either case, multiplied by 60, will give the difference of longitude, in *miles*.

When the difference of longitude *exceeds* 180°, take it from 360°, the remainder, multiplied by 60, will be the diff. long. *in miles*.

EXAMPLE TO CASE (1).

Find the True Course and Distance from Brest to Halifax.

Working of Example.

(*a*)		(*b*)		(*c*)	
Lat. from (Brest)	48° 23′ N.	Lat. Brest	48° 23′ N.	Long. from (Brest)	4° 29′ W.
Lat. to (Halifax)	44 44 N.	Lat. Halifax	44 44 N.	Long. to (Halifax)	63 36 W.
	3 39	2)	93 7		59 7
	60				60
diff. lat.	219*m*.	mid. lat.	46° 33′	diff. long.	3547*m*.

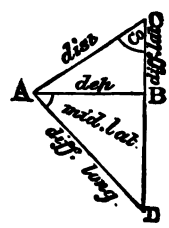
The figure used in this sailing is made up of *two* right-angled triangles, having one side (dep.) common to both.

One triangle is the figure used for *parallel* sailing, and the other is the figure used for *plane* sailing.

In the present case we have,

1st. In the parallel sailing figure, the *mid. lat.* and *diff. long.* are given, from which to find the *dep.*

2nd. In the plane sailing figure, the *diff. lat.* and the *dep.* (just found) are given, to determine the course and distance.



Note.—This figure is drawn out of proportion, in order to show the names of its parts with distinctness.

(1st) in Δ ABD.

By Parallel Sailing.

Given $\left\{\begin{array}{l}\angle BAD = 46^\circ 33' \\ \text{the mid. lat.}^* \\ AD = 3547 \\ \text{the diff. long.}\end{array}\right\}$ to find $\left\{\begin{array}{l}AB \\ \text{the} \\ \text{dep.}\end{array}\right.$

$$\frac{AB}{AD} \text{ or } \frac{\text{dep.}}{\text{diff. long.}} = \text{cos. mid. lat.}^*$$

$\therefore$ dep. = diff. long. cos. mid. lat. (1)

log. diff. long. 3547	3·549861
log. cos. 46° 33′	9·837412
log. dep.	3·387273

$\therefore$ dep. = 2439 miles.

* Here the *middle* latitude between Brest and Halifax is taken as representing the *parallel half-way* between the two latitudes.

By Inspection.

The *dep.* is found from the Traverse Table, by the same method as in Ex. (A) of Parallel Sailing.

(2nd) In Δ ABD.

By Plane Sailing.

Given $\left\{\begin{array}{l}AB = 2439 \text{ dep.} \\ BC = 219 \text{ diff. lat.}\end{array}\right\}$ to find $\left\{\begin{array}{l}\text{co.} \\ \text{and} \\ \text{dist.}\end{array}\right.$

$$\frac{AB}{BC} \text{ or } \frac{\text{dep.}}{\text{diff. lat.}} = \text{tan. co.} \qquad (2)$$

log. dep. 2439	13·387273(+10)
log. diff. lat. 219	2·340444
log. tan. co.	11·046829

$\therefore$ Course = 84° 52′ 15″
or S. 84° 52′ 15″ W.

$$\frac{AC}{BC} \text{ or } \frac{\text{dist.}}{\text{diff. lat.}} = \text{sec. co.}$$

$\therefore$ dist. = diff. lat. sec. co. (3)

log. diff. lat. 219	2·340444
log. sec. co. 84° 52′ 15″	11·048656
log. dist.	3·389100

$\therefore$ dist. = 2450 miles.

By Inspection.

The *dist.* is found from Traverse Table, by the same method as in Ex. 2, of Plane Sailing.

Note.—The course can be found without first obtaining the *departure*, by the formula,

$$\text{tan. co.} = \frac{\text{diff. long. cos. mid. lat.}}{\text{diff. lat.}}$$

CASE (2).

To find the difference of longitude.

(*a*) Write down the lat. of the place from, on the left-hand side of the page, underneath which put the lat. arrived at, and find the difference of latitude.

(*b*) Write down the lat. of the place from, on the right-hand side of the page, underneath which put the lat. arrived at, and find the middle latitude.

EXAMPLE TO CASE (2).

A ship sails N.W. by N., from lat. 44° 15′ N., until she arrives in lat. 46° 15′ N.; find the difference of longitude she has made.

Working of Example.

(*a*)

Lat. from ... 44° 15′ N.
Lat. in ..... 46° 15′ N.

2° 0′
60

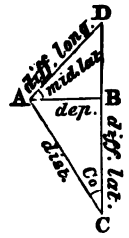
diff. lat. 120 miles.

(*b*)

Lat. from ... 44° 15′ N.
Lat. in ..... 46° 15′ N.

2)90° 30′

mid. lat. 45° 15′



(1st) In Δ ABC.

By Plane Sailing.

Given { co. N. 33° 45′ W. / diff. lat. = 120m. } to find dep.

$\frac{AB}{BC}$ or $\frac{\text{dep.}}{\text{diff. lat.}}$ = tan. co.

∴ dep. = diff. lat. tan. co. (1)

log. diff. lat. 120m.	2·079181
log. tan. co. 33° 45′	9·824893
log. dep.	1·904074

∴ dep. = 80·18m.

By Inspection.

Enter the Traverse Table with the given course, and given diff. lat. in their proper columns, and in the dep. column will be found the dep. required.

(2nd) In Δ ABD.

(See Ex. (B.) Parallel Sailing.)

$\frac{AD}{AB}$ or $\frac{\text{diff. long.}}{\text{dep.}}$ = sec. mid. lat.

∴ diff. long. = dep. sec. mid. lat. (2)

log. dep.	1·904074
log. sec. m. lat. 45° 15′	10·152418
log. diff. long.	2·056492

∴ diff. long. = 113·9m. *westing.*
= 1° 53′ 54″ W.

By Inspection.

The diff. long. is found from the Traverse Table by the method given in Ex. (B), Parallel Sailing.

Note.—The diff. long. can be found without first obtaining the *departure*, by the formula,

diff. long. = diff. lat. tan. co. sec. mid. lat.

MIDDLE LATITUDE SAILING.

EXAMPLES.

1. Find the *true* course and distance from Espichel Cape (Portugal), in lat. 38° 25′ N., and long. 9° 13′ W., to St. Mary's (Azores), in lat. 36° 57′ N., and long. 25° 9′ W.

Ans. True course S. 83° 20′ 45″ W. Dist. 762m.

2. Find the *true* course and distance from Esquimaux Isles (Labrador), in lat. 58° 35′ N., and long. 56° 21′ W., to Bergen (Norway), in lat. 60° 24′ N., and long. 5° 18′ E.

Ans. True course N. 80° 2′ 30″ E. Dist. 2018m.

3. Find the *true* course and distance from Goree (W.C. Africa), in lat. 14° 40′ N., and long. 17° 24′ W., to St. Salvador (Brazil), in lat. 13° 5′ S., and long. 38° 28′ W.

Ans. True course S. 36° 58′ 0″ W. Dist. 2084m.

4. Find the standard *compass* course and distance from Guadaloupe I., in lat. 16° 0′ N., and long. 61° 45′ W., to Sal. I., in lat. 16° 51′ N., and long. 22° 55′ W. Var. 10° W. Dev. 9° E.

Ans. True course N. 88° 42′ 15″ E. Comp. course N. 89° 42′ 15″ E. Dist. 2240m.

5. Find the standard *compass* course and distance from Gough's I., in lat. 40° 19′ S., and long. 9° 44′ W., to Grim Cape, in lat. 40° 40′ S., and long. 144° 42′ E. Var. 4° W. Dev. 12° E.

Ans. True course S. 89° 49′ 45″ E. Comp. course N. 82° 10′ 15″ E. Dist. 7047m.

6. Find the standard *compass* course and distance from Woahoo, in lat. 21° 18′ N., and long. 157° 55′ W., to San Francisco, in lat. 37° 48′ N., and long. 122° 24′ W. Var. 10° E. Dev. 10° E.

Ans. True course N. 61° 53′ E. Comp. course N. 41° 53′ E. Dist. 2101m.

7. A ship in lat. 9° 15′ N. sails E.N.E. until she is in lat. 11° 25′ N.; find her difference of longitude.

Ans. Diff. long. 5° 19′ E.

8. A ship in lat. 10° 17′ S. sails E.N.E. until she is in lat. 8° 55′ S.; find her difference of longitude.

Ans. Diff. long. 3° 21′ E.

9. A ship from Cape Frio (Brazil), in lat. 23° 1′ S., and long. 41° 58′ W., sails S.E. by E. until she is in lat. 25° 25′ S.; find her difference of longitude, and long. arrived at.

Ans. Diff. long. 3° 56′ E. Long. in 38° 2′ W.

MERCATOR'S SAILING.

THIS Sailing is derived from Mercator's projection of the Globe upon a plane, or *flat* surface, in which the degrees of longitude are *everywhere equal;* the degrees of latitude *increase* from the Equator towards the Poles; and the meridians, parallels, and rhumb lines (or courses made by the ship) are *all* represented by straight lines.

The two cases most commonly used in this sailing are,

(1) When the *latitudes* and *longitudes* of two places are given } To find the course and distance between them.

(2) When the *course* between two places and their *latitudes* are given } To find the difference of longitude.

CASE (1).

Find the *True* Course and Distance from one place to another.

(*a*) On the left hand of the page, write down the latitude of the place *from*, underneath which put the latitude of the place *bound to* (or worked for).

(*b*) Write down the corresponding meridional parts of the two latitudes in the middle of the page.

(*c*) On the right hand of the page, write down the longitude of the place *from*, underneath which put the longitude of the place *bound to* (or worked for).

(A) When the latitudes are both of the *same* name, take the less from the greater; but when they are of *contrary* names, take their sum; the result, in either case, multiplied by 60, will give the *true* diff. lat. in *miles*.

(B) Having taken out the meridional parts of the two latitudes from Table y, Inman, take their *difference* when the latitudes are of the *same* name; but their *sum* when the latitudes are of *contrary* names. The result, in either case, will be the *meridional* diff. lat. (mer. diff. lat.) in *miles.*

(C) When the longitudes of the two places are both of the *same* name, take the less from the greater; but when they are of *contrary* names, take their sum; the result, in either case, multiplied by 60, will give the diff. long. in *miles.*

Note.—When the diff. long. *exceeds* 180 degrees, take it from 360 degrees, the remainder must be used as the diff. long. when turned into *miles.*

The difference of latitude (in geographical miles) between two places on the Globe, is called the *true* difference of latitude.

The difference of latitude (in geographical miles) between two places on the Chart, is called the *meridional* difference of latitude.

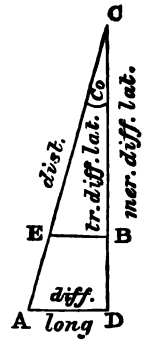
EXAMPLE TO CASE (1).

Find the True Course and Distance from Cape Clear to Porto Santo.

Working of Example.

(*a*)		(*b*)		(*c*)	
Lat. C. Clear (from)	51° 25′ N.	Mer. par.	3609	Long. C. Clear	9° 29′ W.
Lat. P. Santo (to)	33 3 N.	Mer. par.	2103	Long. P. Santo	16 17 W.
	18 22		1506		6 48 W.
	60		mer. diff. lat.		60
True diff. lat.	1102 miles.			diff. long.	408 miles.

Note. — 1st. Work with the triangle CDA, which is formed by side CD, the *mer. diff. lat.*, and side DA the *diff. long.*, finding ∠ C (the course), which is *common* to both triangles. 2nd. Having found the common ∠ C, work with triangle EBC, which is formed by BC, the *true* diff. lat., and CE the *distance* required.



[1st.]

In △ CDA,

Given { mer. diff. lat. 1506m. ; diff. long. 408m. } to find { ∠ C, the course }

$\frac{DA}{DC}$ or $\frac{\text{diff. long.}}{\text{mer. diff. lat.}}$ = tan. co. (1)

log. diff. long. 408 — 12·610660 + 10
log. m. diff. lat. 1506 — 3·177825

log. tan. co. — 9·432835

∴ course = 15° 9′ 30″

or, true course = S. 15° 9′ 30″ W.

[2nd.]

In △ CBE,

Given { true diff. lat. 1102 m. ; true co. 15° 9′ 30″ } to find { distance }

$\frac{CE}{CB}$ or $\frac{\text{dist.}}{\text{tr. diff. lat.}}$ = sec. course

∴ dist. = tr. diff. lat. sec. co. (2)

log. tr. diff. lat. 1102 — 3·042182
log. sec. co. 15° 9′ 30″ — 10·015380

log. dist. — 3·057562

∴ distance = 1141 miles.

Note.—The course must be marked S.....W., because Porto Santo (the place worked for) is to the *southward* and *westward* of Cape Clear—the place *from*.

By Inspection.

With the mer. diff. lat. as *diff. lat.*, and diff. long. as *dep.*, enter the Traverse Table, and find the corresponding *course*. This will be the course required.

Again, with this course, and the true diff. lat. as *diff. lat.*, enter the Traverse Table, and find the corresponding *distance*. This will be the distance required.

Example.—Enter the Traverse Table with mer. diff. lat. 150, 6 (or the nearest number to it) as *diff. lat.*, and the diff. long. 40, 8 (or the nearest number to it) as *dep.*, the corresponding *course* is 15°. This will be course required.

Again, enter the Traverse Table with this *course* 15°, and the true diff. lat. 110, 2 (or the nearest number to it) as *diff. lat.*, and find the corresponding *dist.* 114 (that is, 1140). This will be the distance required. (*See* Note.)

Note.—In the above example, since the numbers which represent the mer. diff. lat., diff. long., and true diff. lat. respectively, are greater than can be found in the Traverse Table, we cut off one figure to the *right* (that is, divide by 10) and seek for these new numbers in the Table; the division thus made, does not effect the *course*, but when finding the *distance* we multiply what the Table gives by 10, which will be the distance required.

Case (2).

To find the Difference of Longitude.

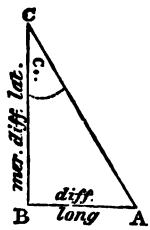
Write down the two latitudes, and take out their mer. parts; and find the *mer. diff. lat.* by the same rule as in Example (1).

Example to Case (2).

A ship from latitude 50° 10′ N., sails S.E. by S. until she is in lat. 48° 10′ N.; find the difference of longitude she has ma

Working of Example.

Lat. from 50° 10′ N.	Mer. part. 3490
Lat. in 48 10 N.	Mer. part. 3306
	Mer. diff. lat. 184 miles.



Given { mer. diff. lat. 184m. / course S. 33° 45′ E. } to find { diff. long.

$$\frac{BA}{BC} \text{ or } \frac{\text{diff. long.}}{\text{mer. diff. lat.}} = \text{tan. co.}$$

$\therefore$ diff. long. = mer. diff. lat. tan. co.

log. mer. diff. lat. 184m.	2·264818
log. tan. co. 33° 45′	9·824893
log. diff. long.	2·089711

$\therefore$ diff. long. = 123 miles, Easting.

60) 123′

or, diff. long. 2° 3′ East.

By Inspection.

With the given *course*, and the mer. diff. lat., as *diff. lat.*, enter the Traverse Table, and the corresponding *dep.* will be the diff. long. required.

Example.—Enter the Traverse Table with course 3 points and the mer. diff. lat. 184 (or the nearest number to it) as *diff. lat.*, and the corresponding *dep.* 123·3 will be the diff. long. required (nearly).

To turn a TRUE Course into a COMPASS Course (or, Course to Steer).

1st. Correct for VARIATION.

Apply the variation to the true course as follows:

Easterly variation to the left.
Westerly variation to the right.

This will give the correct magnetic course.

2nd. Correct for DEVIATION.

Apply the deviation to the correct magnetic course as follows:

Easterly deviation to the left.
Westerly deviation to the right.

The result will give the standard compass course (or, course to steer).

EXAMPLES.

1. The true course from Corunna to Cape Clear is N. 6° W.; what is the standard compass course, or course to steer, when the variation is 20° W., and the deviation 3° E.?

True course	N. 6° W.	
Variation, 20° W.	20	allow to *right* of true course.
Corr. mag. course	N. 14 E.	
Deviation, 3° E.	3	allow to *left* of corr. mag. course.
Stand. comp. course	N. 11 E.	or, course to steer from Corunna to C. Clear.

2. The true course from Mobile to Bermuda is N. 84° E.; what is the standard compass course, or course to steer, when the variation is 5° W., and deviation 10° E.?

True course	N. 84° E.	
Variation, 5° W.	5	allow to *right* of true course.
Corr. mag. course	N. 89 E.	
Deviation, 10° E.	10	allow to *left* of corr. mag. course.
Stand. comp. course	N. 79 E.	or, course to steer from Mobile to Bermuda.

MERCATOR'S SAILING.

EXAMPLES FOR EXERCISE.

1. Find the *true* course and distance from Mobile, in lat. 30° 14′ N. and long. 88° 1′ W., to Bermuda, in lat. 32° 19′ N. and long. 64° 52′ W.—*See* Example, p. 35.

Ans. Course N. 83° 59′ 0″ E. Dist. 1197 miles.

2. Find the *true* course and distance from Bon Cape (Barbary), in lat. 37° 5′ N. and long. 11° 3′ E., to Caprera I. (Mediterranean), in lat. 41° 13′ N. and long. 9° 29′ E.

Ans. Course N. 16° 27′ 0″ W. Dist. 259 miles.

3. Find the *true* course and distance from Broad Haven (Ireland), in lat. 54° 26′ N. and long. 10° 12′ W., to Charlestown (United States), in lat. 32° 46′ N. and long. 79° 55′ W.

Ans. Course S. 66° 25′ W. Dist. 3248 miles.

4. Find the *true* course and distance from Columbia R. (Ent.) in lat. 46° 19′ N. and long. 123° 44′ W., to Combay I. (Indian Ocean), in lat. 7° 49′ S. and long. 123° 51′ E.

Note.—In crossing the meridian of 180° from W. long. to E. long. the course will be westerly.

Ans. Course S. 61° 49′ 30″ W. Dist. 6878 miles.

5. Find the standard *compass* course and distance from Congo Riv., in lat. 6° 5′ S. and long. 12° 15′ E., to St. Salvador, in lat. 13° 5′ S. and long. 38° 28′ W. Var. 5° E. Dev. 7° W.

Ans. True course S. 82° 1′ W. Comp. course S. 84° 1′ W. Dist. 3030 miles.

6. Find the standard *compass* course and distance from Jervis' Island (Torres' Strait), in lat. 9° 55′ S. and long. 142° 10′ E., to Valparaiso (Chile), in lat. 33° 2′ S. and long. 71° 41′ W. Var. 15° E. Dev. 9° E.

Note.—In crossing the meridian of 180° from E. long. to W. long. the course will be easterly.

Ans. True course S. 80° 16′ E. Comp. course N. 75° 44′ E. Dist. 8204 miles.

7. A ship from C. Clear, in lat. 51° 26′ N. and long. 9° 22′ W., sails S.S.W. until she is in lat. 40° 0′ N.; find her difference of longitude.—*See* Example, p. 36.

Ans. Diff. long. 6° 49′ W.

8. A ship from L'Orient (France), in lat. 47° 43′ N. and long. 3° 21′ W., sails N.W. by W. until she is in lat. 49° 15′ N.; find the difference of longitude.

Ans Diff. long. 3° 23′ W.

MISCELLANEOUS EXAMPLES.

1. A point of land in lat. 0° 9′ 40″ N. and long. 50° 10′ W. bore from a ship N. 47° W., distant 22 miles; find the lat. and long. of the ship.

Ans. Lat. 0° 5′ 20″ S. Long. 49° 54′ W.

2. A headland in lat. 0° 2′ 20″ S. and long. 98° 15′ 30″ E. bore from a ship (by standard compass) S.W. by S., distant 21 miles; find the lat. and long. of the ship. Var. 2° E. Dev. 2° W.

Ans. Lat. 0° 15′ 10″ N. Long. 98° 27′ 5″ E.

3. The centre of an island in lat. 0° 10′ S. and long. 66° 15′ W. is observed to bear from a ship S. 20° W. (by standard compass), distant 18 miles; find her lat. and long. Var. 3° E. Dev. 1° W.

Ans. Lat. 6′ 42″ N. Long. 66° 8′ W.

4. From a ship in lat. 50° 5′ N. and long. 2° 7′ W. a point of land was observed to bear N.N.W.$\frac{1}{4}$W., and after sailing W.S.W. 10 miles, it bore N.$\frac{1}{2}$W.; required the lat. and long. of the point.

Ans. Lat. 50° 31′ N. Long. 2° 36′ W.

5. A point of land in lat. 0° 10′ N. and long. 6° 44′ E. is observed to bear north (by compass) 30 miles from a ship bound to St. Helena, in lat. 15° 55′ S. and long. 5° 44′ W. How far is she from her destination, and what course must she steer to reach it? Var. 2° W.

Ans. Course S. 39° 11′ W. Dist. 1173 miles.

6. A rock, not marked on the chart, is discovered bearing N. 20° W. (true) from a ship, and distant 2 miles. Being unable to obtain observations at the time, she sails N. 40° E. 60 miles, and then finds by sights that she is in lat. 10° 30′ S. and long. 108° 20′ W.; required the lat. and long. of the rock.

Ans. Lat. 11° 14′ S. Long. 109° 0′ W.

7. A ship in lat. 0° 7′ 30″ S. and long. 149° 15′ 45″ E. sails N. 17° E. 12 miles, when she grounds on a shoal. From her masthead the sea is observed to break over a rock, bearing

S. 20° E., distant 9 miles; required the lat. and long. of the shoal on which she struck, and also of the rock.

Ans. Lat. of shoal 0° 4′ N. Long. 149° 19′ 15″ E. Lat. of rock 0° 4′ 30″ S. Long. 149° 22′ 20″ E.

8. A ship sailing N.N.E. at the rate of 6 knots, passes a coral reef, the southern extremity of which bears N. 70° W., distant 3 miles. At the end of an hour it is found by observation that the ship is in lat. 11° 30′ S. and long. 134° 25′ W., when the northern extremity bears west, distant 4 miles; required the lat. and long. of the southern extremity of the reef, its length, and the direction in which it lies.

Ans. Lat. of southern extremity 11° 34′ 30″ S. Long. 134° 30′ 15″ W. Length of reef 4·654 miles. Direction in which it lies, N. 13° 52′ E.

9. Wanting to ascertain the breadth of a river, in the entrance to which are several islands; from a cliff on the eastern side I sailed N.N.W. 50 miles, and then running S.S.W. I reached the western shore: required the breadth between these two positions.

Ans. 35·35 miles.

10. Two ships take their departure from the Lizard, in lat. 49° 58′ N., one bound to St. Michael's, which lies 715m. to the south and 745m. to the west; the other bound to Lisbon, which lies 661m. to the south and 215m. to the west, reckoning from the Lizard. They sail in company S.W.½W. 610 miles, and then part. What is the direct course and distance of each ship to her port?

Ans. First ship, S. 43° 7′ E., 375·3m.
Second do., S. 39° 49′ W., 427·1m.

11. A ship, after sailing S.E. by S., 62 leagues from her port, in lat. 29° 16′ N., meets another ship that had run 124 leagues from a place bearing W. by N. of the port the first ship departed from; what was the ship's course, and the distance of the two ports?

Ans. Ship's course, S. 58° 3′ E. Dist. 77·18 leagues.

THE DEAD RECKONING.

IN the Navy, the time in the ship's log-book is reckoned from *midnight*, as civil or common time; the first hour is therefore 1 o'clock in the morning, and the hours are carried on to 12, or *noon*, and then to 12, or *midnight*.

The log, or account of the ship's proceedings, begins and ends at *noon*.

The place of the ship is ascertained every day at NOON, from the *log-account;* besides this, the course and distance made good during the last 24 hours, and the bearing and distance of the port bound to, or an intermediate point of land, with reference to which the course has to be *shaped*, so as to make it, or to avoid it, are to be found.

The work of computation necessary to obtain the above is called a *Day's Work*.

To work a Day's Work.

When a departure from a known point of land has been taken, consider it as a *course* in the *opposite* direction to the bearing.

1. Correct the departure course for deviation and variation, and place it in the Traverse Table, with its distance.

2. Then correct *each* course on the log-board for deviation and variation, also for leeway (if any), and place it with its corresponding distance run, in the Traverse Table.

3. When the ship is known to have been "set" in any given direction by a current (or other cause), consider this "set" as a course, and correct it for variation *only*, and place it with its corresponding distance in the Traverse Table.

4. If the ship is lying-to, take the *middle* point between the direction of her head when she comes up to the wind and when she falls off from it, for the *course*. Allow to this course the *mean* of the deviations upon the two directions of her head, as the deviation; then correct for variation and leeway, as usual.

5. Having thus corrected all the courses, take out the diff. lat. and dep. for each from the Traverse Table (X. Inman).

6. Then add up each column separately, and take the less diff. lat. from the greater, marking the remainder with the name of the greater, which will give the diff. lat. made good.

7. Also, take the less dep. from the greater, marking the remainder with the name of the greater, which will give the dep. made good.

To find the Latitude in, by Dead Reckoning (D.R.).

(*a*) To the latitude by *observation* of the preceding noon, apply the difference of latitude made good, which will give the latitude in by D.R. as follows:—

(1) When the lat. left and diff. lat. are of the *same* name, their *sum* will be the lat. in.*

(2) When the lat. left and diff. lat. are of *contrary* names, take their *difference*, and the remainder will be the lat. in of the name of the greater.*

(*b*) When the latitude was not observed at *noon*, but at some other time, and from it the lat. D.R. has been deduced, the lat. thus obtained should be noted as "brought up," because the lat. by D.R. when employed for comparison with the lat. by *observation* is considered as referred to the beginning of the day, unless the contrary is expressed.

(*c*) When there was no lat. by *observation* at the preceding noon, the lat. D.R. at that time must be used.

To find the Longitude in by Dead Reckoning (D.R.).

To the longitude D.R. of the preceding noon, apply the difference of longitude made good, which will give the long. in by D.R.

If the longitude at the preceding noon has been obtained from observations upon which dependence can be placed, it should be used instead of the long. D.R. to find the long. in.

(1) When the long. left and diff. long. are of the *same* name, their sum will be the long. in.*

* See Notes, pages 12 and 14.

(2) When the long. left and diff. long. are of *contrary* names, take their *difference*, which will be the long. in of the name of the greater.*

Note.—The diff. long. is obtained from the *dep.* made good, either by Ex. (B.) Parallel Sailing, or by Case (2) Middle Latitude Sailing, or by Case (2) Mercator's Sailing.

To find the True Course and Distance made good.

These are found by Mercator's Sailing. (*See* Rule, p. 33.)

To find the True Bearing and Distance.

These are found by Case 1, Middle Latitude Sailing; or by Case 1, Mercator's Sailing.

When the ship is on the same parallel of latitude as the place worked for, the true course will be due E. or due W.; and the distance must be found by Ex. A, Parallel Sailing.

To Shape the Course to Steer.

Having found the true bearing of the place bound to, the course to steer is obtained by applying the variation and deviation. (*See* Rules, p. 37.)

To find the Current (if any).

The course and distance between the place of the ship by D.R. and her place by obs. worked out as in the case of the true bearing and distance (above) will give the set and rate of the current. The *set* being marked as if going from D.R. to observation.

* See Notes, pages 12 and 14

June 14th, at noon, a point of land, in lat. 42° 56′ N., and long. 9° 16′ W., bore by standard compass E. by N., distant 18 miles (deviation 6° W.), afterwards sailed as by the following log account: required the latitude and longitude in at noon of June 15th.

Hours.	Knots.	Tenths.	Standard Compass Courses.	Leeway Points.	Winds.	Deviation.	Remarks.
1	3	6	S.W. b W.	½	N.W. b W.	8° W.	P.M.
2	3	5					
3	4	2					
4	4	4					
5	4	2					
6	4	0	W. b N.	¼	N. b W.	12° W.	Variation allowed, 2¼ points westerly.
7	4	5					
8	5	0					
9	5	2					
10	5	4					
11	6	5					
12	6	3					
1	6	2	E.N.E.	0	North	13° E.	A.M.
2	5	5					
3	5	4					
4	5	0					
5	5	0					
6	4	6					A current set the ship the last 8 hours N. b W. (corr. mag.) 1½ mile an hour.
7	4	0	S.S.E.	¼	East	6° E.	
8	3	3					
9	3	5					
10	3	0					
11	3	2					
12	3	0					

True Courses.—S. 47° W. (dep. co.) 18′. S. 17° W. 19·9′. S. 61° W. 36·9′.
N. 55° E. 31·7′. S. 39° E. 20′. N. 37° W. (current co.) 12′.
Diff. lat. 36·8′ S. Dep. 19·89′ W.
Latitude in, 42° 19′ N. Longitude in, 9° 43′ W.

August 20th, at noon, Ushant, in lat. 48° 28′ N., long. 5° 3′ W., bore S.E. by standard compass, distant 10 miles (deviation 11° W.), afterwards sailed as by the following log account: required the latitude and longitude of the ship on August 21st, at noon.

Hours.	Knots.	Tenths.	Standard Compass Courses.	Leeway Points.	Winds.	Deviation.	Remarks.
1	5	5	West	—	N.N.W.	14° W.	P.M.
2	5	7					
3	6	0					
4	6	2					
5	7	0					
6	7	5					
7	8	3					Variation, 25° westerly
8	7	2	South	—	W. b S.	3° E.	
9	7	0					
10	3	2	S.S.W.*	—	West	4° W.	9·30. Tacked.
	3	0	N.N.W.	½	Ditto	3° W.	
11	5	5					
12	4	2					
1	3	0					A.M.
2	3	2					
3	—		Up S. b E.†	5	S.-westerly	4° E.	2. Hove-to.
4	—		Off S.E. b E.			12° E.	
5	—						
6	—		Up W. b S.†	4	Ditto	12° W.	5. Wore ship.
7	—		Off N.W. b W.			8° W.	
8	—						
9	2	0	W.S.W.	2	South	10° W.	Allow 1 mile an hour for drift, whilst lying-to.
10	3	0					
11	3	6					(See Art. 4, page 41).
12	3	4					

True Courses.—N. 81° W. (dep. co.) 10′. S. 51° W. 46·2′. S. 22° E. 14·2′. S. 6° E. 3·2′. N. 45° W. 18·9′. N. 73° E. 3′. N. 69° W. 3′. S. 55° W. 12′.
Diff. lat. 34·71′ S. Dep. 62·22′ W.
Latitude in, 47° 53′ N. Longitude in, 6° 36′ W.

* When the ship has altered course during the hour, a line is drawn between the two courses made in that interval.

† See (4), page 41.

H.M.S.—From towards

Hours.	Knots.	Tenths.	Standard Compass Courses.	L. W. Points.	Winds.	Force.	Weather Bar. Therm.	Deviation of Stand. Comp.	Remarks.
1									A.M.
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									Noon.

Course and Dist.	Latitude. D.R.	Longitude. D.R.	Var. allowed.	True Bearing and Distance.
Current.	Obs.	Chr.		

1									P.M.
2									
3									
4									
5									
6									
7									
8									
9									
10									
11									
12									Midnight.

Ship's Head by Standard Compass.	Deviation.	Ship's Head by Standard Compass.	Deviation.
	° ′		° ′
NORTH.	1·0 E.	SOUTH.	1·0 E.
N. b E.	2·0	S. b W.	2·0 W.
N.N.E.	3·0	S.S.W.	4·0
N.E. b N.	5·0	S.W. b S.	6·0
N.E.	9·0	S.W.	6·0
N.E. b E.	11·0	S.W. b W.	8·0
E.N.E.	13·0	W.S.W.	10·0
E. b N.	14·0	W. b S.	12·0
EAST.	14·0	WEST.	14·0
E. b S.	15·0	W. b N.	12·0
E.S.E.	13·0	W.N.W.	10·0
S.E. b E.	11·0	N.W. b W.	8·0
S.E.	9·0	N.W.	5·0
S.E. b S.	7·0	N.W. b N.	3·0
S.S.E.	6·0	N.N.W.	2·0
S. b E.	4·0 E.	N. b W.	1·0 W.

Figures to denote the Force of the Wind.

0. Calm.
1. Light air, — Or just sufficient to give steerage way.
2. Light breeze, 3. Gentle breeze, 4. Moderate breeze, — Or that in which a ship, with all sail set, and clean full, would go in smooth water from 1 to 2 knots; 3 to 4 knots; 5 to 6 knots.
5. Fresh breeze, — Royals, &c.
6. Strong breeze, — Single-reefed topsails, and top-gallant sails.
7. Moderate gale, — Or, that to which she could just carry in chase, full and by — Double-reefed topsails, jib, &c.
8. Fresh gale, — Treble-reefed topsails, &c.
9. Strong gale, — Close-reefed topsails and courses.
10. Whole gale, — Or that with which she could scarcely bear close-reefed main-topsail, and reefed foresail.
11. Storm, — Or that which would reduce her to storm staysails.
12. Hurricane, — Or that which no canvas could withstand.

Letters to denote the State of the Weather.

b. Blue sky, whether with clear or hazy atmosphere.
c. Cloudy, but detached opening clouds.
d. Drizzling rain.
f. Foggy. —*f.* Thick fog.
g. Gloomy, dark weather.
h. Hail.
l. Lightning.
m. Misty, hazy atmosphere.
o. Overcast, the whole sky being covered with an impervious cloud.
p. Passing temporary showers.
q. Squally.
r. Rain, continued rain.
s. Snow.
t. Thunder.
u. Ugly threatening appearance of the weather.
v. Visibility of distant objects whether the sky is cloudy or not.
w. Wet, dew.
• under any letter indicates an extraordinary degree.

Thus by combining certain letters we can record the ordinary phenomena of the weather: for example, *bp* would denote blue sky, with passing temporary showers; *qrt* would denote squally weather, with rain and thunder; *gr* would denote gloomy, dark weather, with *very heavy* rain.

NAUTICAL ASTRONOMY.

ON TIME.

As the Earth revolves on its axis from west to east, the Sun *appears* to move in the contrary direction from east to west, making it 12 o'clock, or NOON, at each meridian it successively passes over in its daily course.

Thus, when the Sun is *on* the meridian of Greenwich it is 12 o'clock, *apparent time*, at that place, and the inhabitants of places to the *westward* of Greenwich, having to *wait for the Sun* to make it noon by their clocks, will find their time behind that of Greenwich; whilst those living to the *eastward* of Greenwich, having had the Sun on their meridians *before* the people at Greenwich, will find their time in advance of that of Greenwich.

Places 15° to the westward of Greenwich wait 1 hour, places 30° to the westward of Greenwich wait 2 hours, and so on, for the Sun to cross their meridians after it has passed that of Greenwich.

In a similar manner, places 15° to the eastward of Greenwich have the Sun on their meridian 1 hour before the people at Greenwich; places 30° to the eastward 2 hours, and so on; and their clocks are therefore in advance of the clocks at Greenwich, in proportion to their longitude in time.

Hence, all places to the westward of Greenwich are to be considered *slow* on Greenwich time, whilst all places to the eastward of Greenwich are to be considered *fast* on Greenwich time. Consequently the clocks to the westward will show *less* time, and the clocks to the eastward will show *more* time than the clocks at Greenwich, or than a chronometer set to *Greenwich time*.

EXAMPLE I.

Supposing it is required to find the time at Plymouth, in long. 4° 24′ W., at the instant the clocks at Greenwich are striking 12 at Noon.

Time at Greenwich	12h. om. os. (Noon)
Longitude of Plymouth in time (slow)	0 17 36
Time at Plymouth	11 42 24 A.M.

Since the Sun does not come to the meridian of Plymouth until 17m. 36s. after it has passed the meridian of Greenwich.

EXAMPLE II.

Supposing it is required to find the time at Madras, in long. 80° 14′ 15″ E., when the clocks at Greenwich are striking 12 at Noon.

Time at Greenwich	oh. om. os. (Noon)
Longitude of Madras in time (fast)	5 20 57
Time at Madras	5 20 57 P.M.

Since the Sun passes the meridian of Madras 5h. 20m. 57s. before it comes to the meridian of Greenwich.

It is often necessary to determine the Greenwich time, when the time at another place and the longitude are given.

In this case, by applying the longitude in *time* to the time at the place, the corresponding time at Greenwich can be obtained; remembering that places to the westward are behind, or *slow* of Greenwich, and places to the eastward are in advance, or *fast* of Greenwich, hence the following rules to find the Greenwich date.

THE GREENWICH DATE.

The term Greenwich *date* is here used, being preferable to Greenwich time, because in Nautical Astronomy it is essential to note the day as well as the hour.

The *Civil Day* is that in general use; it begins at midnight and ends at the midnight following. It is divided into two parts of 12 hours each; the first is marked A.M. (before noon), and the latter is marked P.M. (afternoon).

The *Astronomical Day* commences 12 hours after the Civil Day, that is, at NOON, and ends on the following Noon; it is generally reckoned through the 24 hours, from Noon to Noon.

The Noon *of the Civil Day, and the* beginning *of the Astronomical Day take place at the* same *moment.*

The computations in the Nautical Almanac are made for *astronomical* time at the meridian of Greenwich.

TO FIND THE GREENWICH DATE.

RULES.

REDUCE the time at ship to *astronomical* time, by one of the following rules:—

(*a*) When the time at ship is A.M., add 12 to the *hours*, and put the date one day *back;* the result will be the *astronomical time* at ship.

(*b*) When the time at ship is P.M., *omit* the letters P.M., and the ship time will be expressed *astronomically.*

LONGITUDE, **West.**—Turn the longitude into time (by the Table of Haversines), and *add* it to the astronomical time at ship; the sum will be the *Greenwich date*, of the *same* day as the ship date.

Note 1.—If the sum be *greater* than 24 hours, reject 24h.; the remainder will be the Greenwich date, of the day *after* the astronomical *ship* date.

LONGITUDE, **East.**—Turn the longitude into time; *subtract* it from the astronomical time at ship, and the remainder will be the *Greenwich date*, of the *same* day as the astronomical ship date.

Note 2.—If the longitude in time be *greater* than the *astronomical* time, add 24 hours to the ship time, and put the day one back; then *subtract* the longitude in time; the remainder will be the *Greenwich date*, of the day *before* the *ship* date.

EXAMPLE I.

March 10th, at 2h. 50m. P.M. (mean time) at ship, in longitude 60° W.; required the Greenwich date.

Time at ship	2h.	50m.	March 10th (*b*)
Longitude 60°, in time	4	0	W. (add)
Greenwich date	6	50	March 10th.

EXAMPLE II.

September 14th, at 6h. 10m. A.M. apparent time at ship, in longitude 35° E.; required the Greenwich date.

Civil time at ship	6h.	10m.	A.M. Sept. 14th
Add	12	0	
Astronomical time at ship	18	10	Sept. 13th (*a*)
Longitude 35°, in time	2	20	E. (subtract)
Greenwich date	15	50	Sept. 13th.

EXAMPLE *to Note* 1.

Find the Greenwich date corresponding to 9h. 35m. A.M. (civil time) at a ship, on May 1st, in longitude 65° W.

Time at ship	9h.	35m.	A.M. May 1st
Add	12	0	
Astronomical time at ship	21	35	April 30th (*a*)
Longitude 65° W., in time	4	20	(add)
Greenwich date	25	55	April 30th
Subtract	24	0	
Corresponding Greenwich date of the day *after* astronomical ship date (see *Note* 1)	1	55	May 1st

EXAMPLE *to Note* 2.

Find the Greenwich date corresponding to 5h. 25m. P.M. at a ship, on August 1st, in longitude 100° E.

Astronomical time at ship	5h.	25m.	Aug. 1st (*b*)
Add	24	0	
Time at ship	29	25	July 31st (*Note* 2)
Longitude 100° E., in time	6	40	(sub.)
Corresponding Greenwich date of day *before* the astronomical ship date	22	45	July 31st

EXAMPLES.

Find the Greenwich dates corresponding to the following ship dates and longitudes :—

1. Ship, Jan. 4th, at 3h. 30m. P.M., longitude 30° W. and longitude 30° E.

2. Ship, Aug. 9th, at 7h. 20m. P.M., longitude 110° W. and longitude 110° E.

3. Ship, Sept. 14th, at 11h. 30m. P.M., longitude 10° 30′ W. and longitude 10° 30′ E.

4. Ship, Oct. 21st, at 4h. 55m. P.M., longitude 35° 10′ W. and longitude 35° 10′ E.

5. Ship, Nov. 5th, at *midnight*, longitude 16° 55′ W. and longitude 16° 55′ E.

6. Ship, Dec. 2nd, at *midnight*, longitude 115° W. and longitude 115° E.

7. Ship, Feb. 14th, at 7h. 25m. A.M., longitude 100° W. and longitude 100° E.

8. Ship, Aug. 31st, at 9h. 50m. A.M., longitude 60° 15′ W. and longitude 60° 15′ E.

Note.—Midnight denotes 12 hours *after* noon, or 12h. P.M. of the given day.

Answers.

1. G.D. 5h. 30m. Jan. 4th, and 1h. 30m. Jan. 4th.

2. G.D. 14h. 40m. Aug. 9th, and 0h. 0m. (Noon) Aug. 9th.

3. G.D. 12h. 12m. Sept. 14th, and 10h. 48m. Sept. 14th.

4. G.D. 7h. 15m. 40s. Oct. 21st, and 2h. 34m. 20s. Oct. 21st.

5. G.D. 13h. 7m. 40s. Nov. 5th, and 10h. 52m. 20s. Nov. 5th.

6. G.D. 19h. 40m. Dec. 2nd, and 4h. 20m. Dec. 2nd.

7. G.D. 26h. 5m. Feb. 13th (*a*), or, 2h. 5m. Feb. 14th (*Note* 1) } and 12h. 45m. Feb. 13th (*a*).

8. G.D. 25h. 51m. Aug. 30th (*a*), or, 1h. 51m. Aug. 31st (*Note* 1) } and 17h. 49m. Aug. 30th (*a*).

EXAMPLES.

1. A chronometer was 4m. 24s. *fast* on mean time at Portsmouth; find its error on M.T. Greenwich.

The chronometer was	4m.	24s.	fast on M.T. Portsmouth
The long. of Portsmouth is 1° 6′ west, or	4	24	slow on M.T. Greenwich
Error of chronometer is	0	0	

Here it is evident that since the time at Portsmouth is behind that of Greenwich, the chronometer, which was as much *fast* for Portsmouth as Portsmouth is *slow* on Greenwich, can have no error, and shows correct M.T. Greenwich.

2. A chronometer was 22m. 15s. *fast* on M.T. at Cape Town; find its error on Greenwich.

The chronometer was	oh.	22m.	15s.	fast on M.T. Cape Town
The long. of Cape Town is 18° 23′ east, or	1	13	52	fast on M.T. Greenwich
Error of chronometer is	1	36	7	fast on M.T. Greenwich

Here the time at Cape Town, being in *advance* of that at Greenwich, and the chronometer also in advance of Cape Town, the *sum* of these two quantities must be the error of the chronometer *fast* on M.T. Greenwich.

MISCELLANEOUS EXAMPLES.

1. On Jan. 1st, a chronometer was *slow* 3m. 15s. on mean time at Plymouth, in long. 4° 9′ 30″ W.; what was its error on Greenwich mean time?

Ans. Error, 0h. 19m. 53s. slow.

2. On May 25th, a chronometer was *fast* 32m. 30s. on mean time at Cork, in long. 8° 27′ 42″ W.; what was its error on mean time at Greenwich?

Ans. Error, 0h. 1m. 21s. slow.

3. On Nov. 20th, a chronometer was 5h. 18m. 10s. *slow* on mean time at Calcutta, in long. 88° 19′ 12″ E.; what was its error on mean time at Greenwich?

Ans. Error, 0h. 35m. 7s. fast.

4. On Aug. 4th, a chronometer was 4h. 22m. 15s. *fast* on Greenwich mean time; what was its error on the clocks at New York, in long. 74° 1′ W.?

Ans. Error, 9h. 18m. 19s. fast on New York.

5. A chronometer was 2m. 15s. *slow* for mean time at Funchal, Madeira, in long. 16° 54′ 42″ W.; and on arrival at Sta. Cruz, Teneriffe, in long. 16° 14′ 42″ W., it was found to be 1m. 16s. *fast* on the mean time at that place; what was its error on Greenwich mean time?

Ans. Error, 1h. 3m. 43s. slow.

6. A telegram is made from Greenwich to Lisbon, in long. 9° 8′ 15″ W., at 8h. 15m. 30s. A.M., and which takes 19 seconds in transmission; what time should the clocks at Lisbon show when the telegram is received?

Ans. Should show 7h. 39m. 16s. Lisbon time.

7. On Sept. 16th, Cape Hatteras, in long. 75° 30′ W., bore due N. (true) exactly at mean noon, when a chronometer showed 5h. 2m. 0s.; what was its error on mean time at Greenwich?

Ans. No error.

8. May 21st, at mean noon, the east point of Barbadoes (W. Indies), in long. 59° 25′ W., bore S. (true), when a chronometer showed 3h. 56m. 40s.; what was its error on mean time at Greenwich?

Ans. Error, oh. 1m. os. slow.

9. June 20th, at mean noon, the centre of Amsterdam I., in long. 77° 35′ E., bore due S. (true); what time ought a chronometer to show, if it was correct to mean time Greenwich, and of what date?

Ans. 18h. 49m. 40s., June 19th.

10. On Nov. 21st, at about 8h. A.M. mean time at ship, in long. 60° 45′ W., a chronometer showed oh. 4m. 10s., and its error on mean time Greenwich was 14m. 20s. fast; what is the correct mean time by chronometer, and of what date?

Ans. M.T. Chron. 23h. 49m. 50s., Nov. 20th.

11. On Oct. 6th, a chronometer showed 4h. 17m. 54s. the time at ship being about 3h. P.M. and long. 47° 14′ 30″ E., error of chronometer *fast* on Greenwich mean time 5h. 9m. 20s.; what was the correct time by the chronometer, and of what date?

Ans. M.T. Chron. 23h. 8m. 34s., Oct. 5th.

12. Aug. 31st, a chronometer showed oh. 5m. 30s., the time at ship being about 8h. A.M. and the long. 61° 10′ 30″ W., the error of the chronometer 1m. 55s. *slow* on Greenwich mean time; what was the correct time by the chronometer, and of what date?

Ans. M.T. Chron. oh. 7m. 25s., Aug. 31st.

13. At *mean* noon on June 1st, the Lizard, in lat. 49° 58′ N. and long. 5° 12′ W., bore N.N.E. by *compass*, distant 20 miles, var. 23° W., dev. 7° W., when a chronometer showed oh. 24m. 15s. mean time Greenwich; what was its error?

Ans. Error, oh. 3m. 15s. fast.

14. At *app.* noon, on June 16th, Cape Clear, in lat. 51° 26′ N. and long. 9° 22′ W., bore E. by *compass*, distant 16 miles, var. 24° W., dev. 2° E., when a chronometer showed oh. 44m. 20s.; what was its error on mean time at Greenwich?

Ans. Error, oh. 5m. 1s. fast.

15. A set of morning sights was taken for the longitude, the watch used showed 8h. 30m. 10s., and the corresponding time by a chronometer (A) was 8h. 38m. 10s. At 3h. 50m. 14s. P.M., by the same watch, another set of sights was taken; what corresponding time ought the chronometer to show, supposing neither the rate of the watch nor of the chronometer had altered in the interval?

Ans. 3h. 58m. 14s. P.M.

16. At 9h. 6m. 22s. A.M., a watch was *fast* 1h. 2m. 10s. on a chronometer (C). At 3h. 6m. 22s. P.M. a set of sights was taken, by the same watch, the rate of which was 24s. *losing;* what was the corresponding time by the chronometer, if it had no appreciable rate?

Ans. Time, 2h. 4m. 6s. P.M.

17. At 8h. P.M. a watch is exactly right for ship mean time; by 8h. A.M. on the following morning we have changed our longitude 72 miles to the *eastward* of our former position; how much must I alter the watch to set it to ship mean time?

Ans. To be put forward 4m. 48s.

18. At noon, in lat. 30° S., my watch is set right to mean time at ship, and by the following *noon* I have made a S.W. (true) course, distance 120 miles; what is the error of my watch on mean time at ship?

Ans. Error, 6m. 28s. fast.

19. On June 1st, a chronometer (A) is 15m. fast on M. T. G. and daily rate 3s. losing; when a chronometer (B) is found to be 1m. fast of (A). On June 21st they are again compared, and (B) is found to be 1m. slow on (A); what is B's daily rate, and error on M. T. G.

Ans. Daily rate of (B), 9s. losing.
Error on M. T. G., 13m. fast.

TO CORRECT THE RIGHT ASCENSION OF MEAN SUN.

(Sidereal Time.)

RULE.

HAVING found the Greenwich date from the ship mean time and longitude, take from the Nautical Almanac (page II. of the month) the R.A. mean sun (given under the heading "Sidereal Time"), for noon of the Greenwich date.

Then from Table (o.) I. of Inman, page 10,* or from the Table of "Time Equivalents" in the Nautical Almanac take the increase of sidereal time (R.A. mean sun) corresponding to the hours and minutes of the *Greenwich date*, which *add* to the sidereal time taken out; the sum will be the sidereal time (R.A. mean sun) required.

EXAMPLE.

On Sept. 7th, at 9h. 50m. P.M. mean time at ship, in long. 34° 28′ E.; find the right ascension of mean sun.

Green. Date.					Right Ascension Mean Sun (or Sidereal Time).			
	h.	m.	s.			h.	m.	s.
Time at ship	9	50	0	Sept. 7th	Sid. time, noon, Sept. 7th	11	5	20·34
Long. in time	2	17	52	E.	Correction, Tab. (o.) I.:			
					For 7h.	+	1	8·99
Green. date	7	32	8	Sept. 7th	For 32m.	+	0	5·26
					For 8s.	+	0	0·02
					R.A. mean sun	11	6	34·61
					(Or sidereal time)			

EXAMPLES.

1. Find the right ascension of mean sun, on Sept. 10th, at 7h. 30m. M. T. Green.
2. On Sept. 14th, at 19h. 44m. 35s. M. T. G.
3. On Sept. 16th, at 2h. 14m. M. T. G.
4. On Sept. 20th, at midnight, in long. 60° W., for M. T. G.
5. On Sept. 22nd, at 7h. 50m. A.M., in long. 42° E.

ANSWERS.

	h.	m.	s.
1.	11	18	24
2.	11	36	11
3.	11	41	11
4.	11	59	13
5.	12	3	20

TO FIND WHAT

STARS WILL PASS THE MERIDIAN

BETWEEN ANY TWO GIVEN SHIP MEAN TIMES.*

RULE.

FOR each given ship mean time get a Greenwich date.

Then take from the Nautical Almanac the right ascension of mean sun and correct it for the hours and minutes of each Greenwich date, by Table (o.) I. Inman.

Add together each ship mean time expressed astronomically, and the corresponding R.A. mean sun (rejecting 24 hours from either sum if greater than 24 hours), which will give for results two separate right ascensions of meridian.

Then look in the Catalogue of Fixed Stars in the Nautical Almanac, and all those whose *right ascensions* are found to be between the two results (above), taken in order as limits, will pass the meridian between the given ship times.

Note.—The principal stars used for finding the latitude are given in the Nautical Almanac, pp. 336 to 385.

EXAMPLE.

What principal bright stars will pass the meridian of a ship in longitude 54° E. between 8h. and 11h. P.M. on Sept. 17th?

1st time at ship	8h.	om.	Sept. 17th	2nd time at ship	11h.	om.	Sept 17th
Long. in time	3	36	E.	Long. in time	3	36	E.
1st Green. date	4	24	Sept. 17th	2nd Green. date	7	24	Sept. 17th

* This rule will enable an observer to ascertain whether a star of sufficient magnitude will pass the meridian about the time at which he may require the latitude.

	h.	m.	s.			h.	m.	s.
R.A.M. sun 17th Sept.	11	44	45·87			11	44	45·87
Corr. Tab. (o.) I, Inman:								
For 4h.	+	0	39·42	For 7h.		+	1	8·99
For 24m.	+	0	3·94	For 24m.		+	0	3·94
	11	45	29·23			11	45	58·80
1st time at ship ...	8	0	0	2d time at ship ...		11	0	0
1st R.A. meridian ...	19	45	29	2d R.A. meridian		22	45	59

Then the stars will be those in the Catalogue* whose right ascensions are *greater* than the 1st R.A. meridian, and *less* than the 2nd R.A. meridian, or, from *α* Pavonis to *α* Gruis (inclusive).

EXAMPLES.

1. What bright stars will pass the meridian of a ship in longitude 75° W. between 11h. P.M., Sept. 15th, and 1h. A.M. on Sept. 16th?

Ans. From Fomalhaut to *β* Ceti.

2. What bright stars will pass the meridian of Portsmouth, in long. 1° 6′ W., on Sept. 7th, between 8h. and 11h. P.M.?

Ans. From *α* Aquilæ to *α* Gruis.

3. What bright stars will pass the meridian of Greenwich between 7h. and 9h. P.M., Sept. 20th?

Ans. From *α* Aquilæ to *α* Cygni.

4. What bright stars will pass the meridian of Paris, in long. 2° 20′ E., between 2h. and 5h. A.M., Sept. 20th?

Ans. From *α* Arietis to *α* Tauri.

5. What bright stars will pass the meridian of Cape Blanco (Patagonia), in long. 65° 44′ W., between 8h. and 10h. P.M., Sept. 21st?

Ans. From *α* Pavonis to *α* Gruis.

* Stars of less than the 3rd magnitude are omitted in the Catalogue appended to this book.

TO FIND THE SHIP MEAN TIME WHEN A STAR IS ON THE MERIDIAN.

RULE.

Take from the Nautical Almanac the right ascension of the star, and also the right ascension of the mean sun.

Then, from the right ascension of the star (increased if necessary by 24 hours), subtract the right ascension of the mean sun; the remainder will be the ship mean time when the star is on the meridian.

Note.—Where the mean time of passage is required very accurately, the right ascension of the mean sun must be corrected for the *hours* and *minutes* of the Greenwich date (by Table (o.) I, Inman).

EXAMPLE.

At what mean time (nearly) will the star *α Aquilæ* pass the meridian of a ship in longitude 35° 30′ E. on Sept. 7th?

	h.	m.	s.	
R.A. α Aquilæ, Sept. 7th	19	44	15	
R.A. mean sun, Sept. 7th	11	5	20	
Mean time of passage at ship (nearly)	8	38	55	Sept. 7th

The same (accurately, see *Note*).

	h.	m.			h.	m.	s.
Time at ship	8	39	Sept. 7th	R.A. mean sun, Sept. 7th	11	5	20·34
Long. in time	2	22	E.	Corr. Tab. (o.) I, Inman:			
				For 6h.	+ 0		59·14
Greenwich date	6	17	Sept. 7th	For 17m.	+ 0		2·79
				R.A. mean sun	11	6	22·27

	h.	m.	s.
R.A. α Aquilæ	19	44	15
R.A. mean sun	11	6	22
Mean time of passage* ...	8	37	53

* Having found by the last Rule what stars will pass the meridian between any two given ship times, we can ascertain as above at what moment one of those stars will be on the meridian, and therefore suitable for determining the latitude by meridian altitude.

EXAMPLES.

1. At a ship in longitude 45° W., it is required to find at what time the star *a Aquilæ* will pass the meridian on Sept. 10th.

Ans. 8h. 25m. 2s.

2. At what time will *a Cephei* be on the meridian of a ship in longitude 70° E. on Sept. 8th?

Ans. 10h. 5m. 13s.

3. At what time will *a Scorpii* pass the meridian of a ship in longitude 150° E. on Sept. 7th?

Ans. 5h. 12m. 32s.

4. At what time will *Spica* pass the meridian of a ship in longitude 179° 59′ W. on Sept. 16th?

Ans. 1h. 35m. 4s.

Note.—If the zenith distance be required, take the difference between the declin. of the body and the latitude when the names are alike; marking it with the common name if the declin. be the greater, and with a contrary name if the declin. be the less. When the lat. and declin. are unlike, take their sum, and put to it the name of the declin. The result will be the meridian zenith distance, N. or S. of the zenith.

TO FIND THE LATITUDE BY THE MERIDIAN ALTITUDE OF A FIXED STAR.

RULE.

CORRECT the observed altitude for { *Index error*, *Dip*, *Refraction*, } which will give the true altitude.

Subtract the true altitude from 90°, and the remainder will be the *meridian zenith distance*, which mark N. or S. according as the observer is N. or S. of the star.

Then, from the Naut. Almanac (Table of Fixed Stars) take out the declin. for the day *nearest* to that at ship; mark it with its proper name N. or S., and put it underneath the zen. dist.

If the zen. dist. and declin. are of *like* names, their *sum* will be the latitude, of the name of either.

If the zen. dist. and declin. are of *contrary* names, their *difference* will be the latitude, of the name of the *greater*.

EXAMPLE.

January 15th, the obs. mer. alt. Markab. was 51° 10′ 15″ (Z.N.), I.E. −1′ 10″, dip 21 ft.; required the latitude.

Obs. alt.	51°	10′	15″	(Zen. N.)
Index error		− 1	10	
	51	9	5	
Dip		− 4	31	(Tab. e.)
	51	4	34	
Ref.		− 0	47	(Tab. n.)
True alt.	51	3	47	
	90	0	0	
Zen. dist.	38	56	13	N.
Declin.	14	29	6	N.
Latitude	53	25	19	N.

TO FIND THE LATITUDE BY THE MERIDIAN ALTITUDE OF A FIXED STAR UNDER THE POLE.

RULES.

(A) CORRECT the observed altitude for { *Index error*, *Dip*, *Refraction*, } which will give the true altitude.

Then *add* 90° to the true altitude, and from the sum *subtract* the star's declin.; the remainder will be the latitude, *always* of the same *name* as the Pole.

Or,

(B) Correct the observed altitude for { *Index error*, *Dip*, *Refraction*, } which will give the true altitude.

Then *add* the star's polar distance (90° −declin.) to the true altitude, the sum will be the latitude, of the same *name* as the Pole.

EXAMPLE (A).

June 20th, the obs. mer. alt. α Cephei (under N. Pole), was 30° 10′ 30″, I.E. +2′ 15″, height of eye 15 ft.; required latitude.

Obs. alt.	30°	10′	30″	
Index error	+	2	15	
	30	12	45	
Dip	−	3	49	(Tab. e.)
	30	8	56	
Ref.	−	1	40	(Tab. n.)
True alt.	30	7	16	
Add	90	0	0	
	120	7	16	
Declin.	62	1	6	N. (p. 379, N.A.)
Latitude	58	6	10	N.

EXAMPLE (B).

True alt.	30°	7′	16″	
Pol. dist.	27	58	54	
Latitude	58	6	10	N.

TO CORRECT THE SUN'S DECLINATION.

RULE.

BY HOURLY DIFFERENCES.

HAVING found the Greenwich date from the ship time and longitude, take out of the Nautical Almanac the sun's declin. for apparent or mean noon of the Greenwich date (as required by the Question); also take from page I. the difference for 1h.

Multiply this *diff.* by the hours and decimal parts of an hour (*Note* 2) of the *Green. date:* reduce the product to minutes and seconds and *add* it to the declin. (taken out) when the declin. is *increasing*, or *subtract* it from the declin. (taken out) when the declin. is *decreasing:* the result will be the sun's declin. at the given *Greenwich time.*

Note 1.—The correction for hourly diff. is added to or subtracted from the declin. when it (*the declin.*) is increasing or decreasing, and not because the *hourly diff.* increases or decreases.

Note 2.—To find the decimal parts of an hour, divide the given minutes by 6. Thus, 7h. 42m. = $7^{h}{\cdot}7$, and 19h. 20m. = $19^{h}{\cdot}3$ (nearly).

Note 3.—In the case where the *correction* (when subtractive) *exceeds* the declin. for the day taken out, the *difference* between the declin. and the correction will give the declin. required, but which must then be marked with a *contrary* name to that given in the Nautical Almanac.

TO FIND THE LATITUDE BY THE MERIDIAN ALTITUDE OF THE SUN.

RULES.

(A) *In West Longitude.*

1. Turn the longitude of the ship into time, and *add* it to oh. om. (see *Note*). The sum, putting before it the day of the month at *ship*, will be the Greenwich date.

(B) *In East Longitude.*

2. Turn the longitude of the ship into time, and *subtract* it from 24h. om. (see *Note*). The remainder, putting before it the day of the month *preceding* that at *ship*, will be the Greenwich date.

3. Take from the Nautical Almanac (page I. of the month) the declin. at apparent noon, and also the hourly diff.; multiply the hourly diff. by the hours and decimal parts of an hour of the Greenwich date, and apply the correction thus obtained to the declin., which mark with its proper name, N. or S., as in the preceding Rule.

4. Also take out the sun's semidiameter from the Nautical Almanac (page II. of the month), for noon of the Greenwich date.

5. Correct the observed meridian altitude,

For { Index error, + *off the arc*, or − *on the arc.*
Dip (Tab. e. Inman) *always* −
Ref. − Par. (Tab. m. Inman) *always* −
Semidiameter + L.L. − U.L.

which will give the true altitude of sun's centre.

Note.—When finding the *Greenwich date* in this question, it is usual to consider NOON (the time of observation), as oh. om. of the *day at ship*; or as 24h. om. of the *day preceding* that at *ship*, both being the same moment of time.

6. *Subtract* the true altitude from 90°, which will give the zenith distance; mark this N. or S., according as the *observer* is to the N. or S. of the sun.

7. Under the zenith distance place the declin. (both being marked with their proper names, N. or S.).

When the names are *alike* take the *sum*, which will give the latitude, of the name of either zenith distance or declin.

8. When the zenith distance and declin. are of *contrary* names, take the less from the greater for the latitude, which will be of the name of the greater.

EXAMPLE.

Sept. 28th, in long. 45° W., the obs. mer. alt. of the sun's L.L. was 47° 10′ 30″ (zen. N.). Index error −2′ 40″, height of the eye 18 feet; required the latitude.

(1) Green. Date.	h.	m.		(3) Declin., page I., N.A.				(3) Change, p. I., N.A.	
Time at Ship (NOON)* or	0	0	Sp. 28	Dec. 28 Sept.	2°	1′	54″ S.	Hourly diff.	58″·46
Long. in time	3	0	W.	Corr. for 3h	+	2	55	× by G.D.	3
Green. date	3	0	Sp. 28	Reduced dec.	2	4	49 S.		6,0)17,5″·38
* (See *Note.*)								Corr. for 3h. G.D.	2′ 55″

To find the Latitude.

(5)	Obs. alt. L.L.	47°	10′	30″	(Zen. N.)
	In. error		− 2	40	
		47	7	50	
	Dip 18 ft.		− 4	11	(Tab. e.)
	App. alt. L.L.	47	3	39	
	Ref.		− 0	48	(Tab. m.)
	True alt. L.L.	47	2	51	
(4)	Semid.		+16	1	N.A. p. II. month.
	True alt. centre	47	18	52	
		90			
(6)	Zen. dist.	42	41	8	N.
	Declin.	2	4	49	S.
(8)	Latitude	40	36	19	N.

LATITUDE BY POLE STAR (POLARIS).

RULE.

Nautical Almanac Method.

1. WITH the mean time at ship and longitude, get the Greenwich date.

2. For this date take out the *sidereal time* (R.A. mean sun) (page II. of the month in the Nautical Almanac).

Then enter Table (o.) I, Inman, page 10,* and take out the increase for the hours and minutes of the Greenwich date, which correction must be *added* to the sidereal time.

3. To the sidereal time (R.A. mean sun) thus corrected, *add* the *astronomical time at ship*. Call the result the right ascension of meridian (or sidereal time *of observation*).

Note.—When the *sum* of the sidereal time and *astronomical* time at ship exceeds 24h., reject 24h. and use the remainder in looking out the 1st, 2nd, and 3rd corrections from the Nautical Almanac.

4. Correct the observed altitude for index error, dip, and refraction, and get the true altitude.

5. From the true altitude subtract 1′. This will give the reduced altitude.

6. Enter Table I. (page 499 of Nautical Almanac) with the *sidereal time of observation*, on its proper side of the column of *Correction*. Take out the *first* correction, which must be marked with the algebraic sign *next* to the column "sidereal time."

7. Apply this correction to the reduced altitude, according to the sign + or −. This will be the *approximate* latitude.

8. Then enter Table II. (pages 500, 501 of Nautical Almanac) with the *nearest* degree to the true altitude at the *top*, and sidereal time of observation at the *side*. Take out the *second* correction, which is *always to be added.*

9. Then enter Table III. (pages 500, 501 of Nautical Almanac) with the month at the *top*, and hour of sidereal time of observation at the *side*, take out the *third* correction, which is *always to be added.*

10. Add these two corrections to the *approximate* latitude, the sum will give the latitude (*always north*).

* By *exact* interpolation of the Tables, a difference of a few seconds in the latitude would have resulted: but at sea, for all practical purposes, *seconds* may be omitted.

EXAMPLE—A.M. TIME AT SHIP.

Sept. 4th, at 4h. 52m. A.M. (mean time) in long. 60° E., the obs. alt. of the Pole Star was 20° 10′ 40″, index error −2′ 15″, height of the eye 15 feet; required the latitude.

(1) Green. Date.	(2) Sidereal Time (or R.A. Mean Sun).	(3) R.A. Meridian (or Sid. Time of Obs.).
h. m. Time at ship 4 52 A.M., Sept. 4th 12 *Ast. T. ship.* 16 52 Sept. 3rd Long. in time 4 0 E. Green. date 12 52 Sept. 3rd	h. m. s. R.A.M. sun 10 49 34·14 Corr. Tab. (o.) I: For 12h. + 1 58·28 For 52m. + 0 8·54 R.A.M. sun 10 51 40·96 (Or, sid. time.)	h. m. s. R.A.M. sun 10 51 41 *Ast. T. ship* 16 52 0 27 43 41 Sub. 24 R.A. merid. 3 43 41 (Or sid. time of obs.) (See *Note* to 3.)

Computation of the Latitude.

(4)	Obs. alt.	20° 10′ 40″		
	Index error	− 2 15		
		20 8 25		
	Dip, 15 feet	− 3 49		(Tab. e.)
	App. alt.	20 4 36		
	Ref.	− 2 39		(Tab. n.)
	True alt.	20 1 57		
(5)	Subtract	− 1 0		
	Reduced alt.	20 0 57		
(6)	Sid. time of obs. 3h. 40m. from Tab. I., N.A., page 499, gives *first* correction	− 1 5 45		
(7)	Approximate lat.	18 55 12		
(8)	True alt. 20° 0′ at top of Tab. II., N.A., page 500, and sid. time obs. 3h. 30m. at side, gives *second* correction	+ 0 10		
(9)	Month (Sept. 4th) taken from Tab. III., p. 501, as Sept. 1st, and 4h. 0m. sid. time of obs., gives *third* correction	+ 0 28		
(10)	Latitude*	18 55 50		North.

* By *exact* interpolation of the Tables, a difference of a few seconds in the latitude would have resulted: but at sea, for all practical purposes, *seconds* may be omitted.

VARIATION BY AMPLITUDE (SUN).

RULE.

1. WITH the time at ship and longitude, get a Greenwich date.

2. For this date take out the sun's declination from the Nautical Almanac (page I. or II. of the month), according as apparent or mean time is given, and correct it.

3. Then, sin. true amp. = sin. dec. × sec. lat.

or,

Add together the log. sine of the declin., and the log. secant of the lat., the sum (rejecting the tens in the index), will be the log. sine of the true amplitude.

4. Write E. *before* the true amplitude when the body is *rising*.

5. Write W. *before* the true amplitude when the body is *setting*.

6. Write N. *after* the true amplitude when the declin. is N.

7. Write S. *after* the true amplitude when the declin. is S.

8. Then, correct the compass amplitude for deviation, by allowing it to the *right* when easterly, and to the *left* when westerly. The result will be the *correct magnetic amplitude*. (See *Note*.)

9. Place the *correct magnetic* amplitude under the true amplitude.

10. When they are both reckoned *towards* N., or both reckoned *towards* S., their *difference* will be the *variation*.

11. When one is reckoned *towards* N., and the other *towards* S., their *sum* will be the *variation*.

12. When the *true* amplitude is to the *right* of the magnetic amplitude, the variation will be *easterly*.

(Supposing yourself at the CENTRE of the compass.)

13. When the true amplitude is to the *left* of the magnetic amplitude, the variation will be *westerly*.

(Supposing yourself at the CENTRE of the compass.)

Note.—Should the *correct magnetic amplitude* not be reckoned *from* the same point as the true amplitude (that is *from* E. or W.), take the correct magnetic amplitude from 90° to make it correspond to the true amplitude.

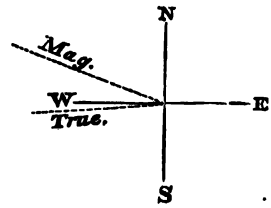
EXAMPLE.

September 23rd, at about 6h. om. P.M. *mean time*, at ship, in lat. 40° 9′ 15″ S., and long. 52° 30′ E., the sun's setting amplitude* by compass was N. 60° 10′ W., deviation 8° 40′ W.; required the variation.

(1) Green. date.		(2) Declin. page II. N.A.	(2) Change, p. I. N. A.
	h. m.		
Time at ship	6 0 Sept. 23rd	Dec. Sept. 23rd 0° 5′ 2″ S.	Hourly diff. 58″·48
Long. in time	3 30 E.	Correction for 2h. 30m. + 2 26	× by G.D. 2 ·5
Green. date	2 30 Sept. 23rd	Declin. 0 7 28 S.	29240
			11696
			6,0)14,6·200
			Corr. 2′ 26″

(3) Sin. true amp. = sin. dec. sec. lat.

Sin dec. 0° 7′ 28″ S. ... 7·336852†
Sec. lat. 40° 9′ 15″ ...10·116729
Sin. true amp. 7·453581†



True amp. (5) W. 0° 9′ 46″ S. (7) Compass amp. N. 60° 10′ W.
(9) Corr. mag. amp. W. 21 10 0 N. Deviation 8 40 left.

(13) Variation 21 19 46 W. N. 68 50 W.
90 (see *Note*)
(8) Corr. mag. amp. W. 21 10 N.

* To find the time of sunset or sunrise by Inspection. Table (z) Inman :—

By the time amplitude is meant the time the sun rises before or after 6 A.M., or sets before or after 6 P.M. When the lat. and declin. are both N. or both S., the sun rises so much *before* 6 A.M., or sets so much *after* 6 P.M. When the lat. and declin. are one N. and the other S., the sun rises so much *after* 6 A.M., and sets so much *before* 6 P.M.

Enter the Table with declin. at top and latitude at the side, and take out the corresponding time, which *add* to or *subtract* from 6 hours, apparent time, according to the above explanation.

† Using Table (s. Inman).

VARIATION BY AZIMUTH (SUN).

RULE.

1. WITH the ship mean time and longitude get the Green. date.

2. For this date take out the sun's declination from the Nautical Almanac (page II. of the month), and correct it. Take out also the sun's semid.

3. Find the sun's polar distance.

(*a*) By subtracting the declin. from 90°, when the latitude and declin. are of the *same* names.

(*b*) By adding 90° to the declin., when the latitude and declin. are of *contrary* names.

4. Correct the observed altitude for index error, dip, refraction, and semid., and thus get the true altitude of the sun's centre.

5. Under the latitude put the true altitude and take the *difference*. Under this difference put the polar distance, and take the *sum* and *difference*.

Then, take out the log. *secant* of the latitude.
log. *secant* of the true altitude.
½ log. *havers.* of the sum.
½ log. *havers.* of the difference.

Add these 4 logs. together (reject the tens in the index) and look it out as a *haversine* (in degrees).

This will give the *true* bearing, to be marked as follows:—

6. Write N. *before* the degrees of the true bearing when the latitude is N.

7. Write S. *before* the degrees of the true bearing when the latitude is S.

8. Write E. *after* the degrees of the true bearing when the body is east of meridian.

9. Write W. *after* the degrees of the true bearing when the body is west of meridian.

10. Correct the *compass* bearing for deviation, allowing it to the *right* when the deviation is *easterly*, and to the *left* when the deviation is westerly. Call the result the *correct magnetic* bearing.

11. Under the true bearing put the correct magnetic bearing, and take the *difference.*

This will be the *variation* of the compass.

12. *East*, when the *true* bearing is to the *right* of the magnetic bearing, supposing yourself at the CENTRE of the compass.

13. *West*, when the *true* bearing is to the *left* of the magnetic bearing, supposing yourself at the CENTRE of the compass.

(A) When observing the azimuth of a heavenly body with the compass, the bearing should always be reckoned from North or from South, never from East or from West; and remember that the *magnetic* azimuth must be reckoned from the *same* point (N. or S.) as the *true* azimuth. To effect this, the compass bearing will sometimes have to be subtracted from 180°.

(B) It may occasionally happen that a question is given in which the magnetic bearing is reckoned *towards* the opposite point to that *from* which the true bearing is reckoned, in which case 90° must be *added* to the magnetic bearing.

(C) When the magnetic bearing is reckoned *towards* the same point as that *from* which the true bearing is reckoned, the magnetic bearing must be *subtracted* from 90°.

Note.—By observing the sun's azimuth, and comparing the variation of the compass so determined with the *known* variation at the place of observation, the deviation on the course (at the time the azimuth is taken) can at once be ascertained; and therefore the deviations for *any given directions* of the ship's head.

EXAMPLE.

September 24th, at 3h. 10m. P.M. (mean time) in lat. 10° 15′ N., and long. 168° 0′ E., the obs. alt. sun's L.L. was 39° 28′ 0″, index error + 1′ 15″, height of the eye 18 feet, when it bore by standard compass S. 84° 20′ W., dev. 13° W.; required the variation.

(1) Green. date.			(2) Declin. page II., N. A.		(2) Change, p. I., N. A.	
	h. m.					
Time at ship	3 10	Sept. 24th	Dec. Sept. 23rd	0° 5′ 2″ S.	Hourly diff.	58″·48
	24		Corr.	+15 36	× by G.D.	16
	27 10	Sept. 23rd	Declin.	0 20 38 S.		35088
Long. in time	11 12	E.		90		5848
Green. date	15 58	Sept. 23rd	Pol. dist. (3b)	90 20 38		6ρ)93,568
						Corr. 15′ 36″

(4) *To find the True Altitude.*

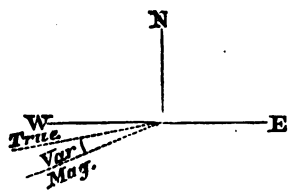
Obs. alt. L.L.	39° 28′	0″
Index error	+ 1	15
	39 29	15
Dip	— 4	11
	39 25	4
Ref.	— 1	4
	39 24	0
Semid.	+15	59
True alt. centre	39 39	59

(5) *To compute the True Bearing.*

Lat.	10° 15′	0″	Sec.	0·006987
True alt.	39 39	59	Sec.	0·113638
	29 24	59		
Pol. dist.	90 20	38		
Sum	119 45	37	½ Hav.	4·937000
Diff.	60 55	39	½ Hav.	4·705013
			Haversine	9·762638

True bearing (6) N. 99° 5′ 0″ W. (9)
(11) Corr. mag. bearing N. 108 40 0 W.

Variation 9 35 0 E. (12)



(10) Compass bearing S. 84° 20′ 0″ W.
Deviation 13 0 0 *left.*

S. 71 20 0 W.
180

Corr. mag. bearing, see (*A*) } N. 108 40 0 W.

TO CORRECT THE EQUATION OF TIME.

WITH the time at ship and longitude get a Greenwich date. For this date take out the equation of time from page II. of the month in the Nautical Almanac; also from page I. take out the *hourly difference.*

Multiply the *hourly difference* by the hours and decimal parts of an hour of the Greenwich date, which will give the *correction.*

Add this correction to the equation of time taken out, when the equation of time is *increasing.*

Subtract this correction from the equation of time taken out, when the equation of time is *decreasing.*

The sum or remainder will be the equation of time required.

Note 1.—When the *correction* exceeds the equation of time taken out from the Nautical Almanac, the *difference* between the correction and the equation of time will be the equation of time required.

Note 2.—Since the equation of time is used either for turning mean into *apparent* time, or for turning apparent into *mean* time, it will be necessary to attend to the precept given at the *head* of the column of equation of time.

Note 3.—When a line ——— is placed at the heading of the column, a similar one comes between two following days in the column itself; in which case, it means that *all* the equations of time *above* the line are to be applied according to the direction *above* the line in the heading; and *all* the equations of time *below* the line must be applied as directed *below* the line in the heading.

Note 4.—The line ——— is placed between those two days on which the equation of time *changes* from additive to subtractive, and *vice versâ*, showing that at some particular moment in the interval the equation of time has become 0m. 0s.

LONGITUDE BY CHRONOMETER (SUN).

RULE.

1. WITH the *estimated* mean time at ship and the longitude by *account*, get the approximate (or rough) Greenwich date.*

2. To the time *shown* by chronometer, apply its error on mean time at Greenwich.†

(*a*) When the error is *slow*, add it to the time shown by chronometer.

(*b*) When the error is *fast*, subtract it from the time shown by chronometer.‡

3. Then multiply the *daily rate* by the number of days and decimal parts of a day elapsed between the time at which the rate was *last* ascertained, and the *Greenwich* time.

Turn this product into minutes and seconds.

(*a*) *Adding* the result to the chronometer time (found in 2 above) when the rate is *losing*.

(*b*) *Subtracting* the result from the chronometer time (found in 2 above) when the rate is *gaining*.

(*o*) This will give the Greenwich *mean time by the chronometer*.§

(*p*) Affix to the mean time by chronometer the day of the month indicated by the approximate Greenwich date.‖

4. Take out the sun's declination and equation of time from page II. of the month in the Nautical Almanac, and correct them for the mean time at Greenwich by the *chronometer*.

5. Mark the corrected equation of time "*added to*," or "*subtracted from*," apparent time, as directed by the heading of the column of Eq. Time in page I. of the month, in the Nautical Almanac.

* The approximate Greenwich time is not used in correcting the declination or equation of time.

† At sea, it is customary to *combine* the error and rate, and apply the result to the time *shown* by the chronometer.

‡ When the error is *fast*, and greater than the time shown by the chronometer, it will be necessary (before subtracting the error) to increase the chron. time by 12 or 24 hours, according as the *Greenwich* date shows the time to be near *midnight*, or near *noon*.

§ As the chronometer mean time is not always the proper time at *Greenwich*, it will occasionally require to be brought to that time by adding or subtracting 12 hours. When to do this will be known by comparing the chron. time with the *Greenwich date*, with which it must be made to agree.

‖ When the *hour* shown by the chronometer happens to be on one side of *noon*, and the hour of the Greenwich date on the other, a consideration of the circumstances of the case will decide whether the chron. should be a day in *advance* of or *behind* the approx. Greenwich date.

6. Correct the observed altitude for index error, dip (ref. —par.) and semidiameter; get the true altitude, and find the *zenith distance.**

7. To compute the *apparent time at ship* (sun's hour angle). Under the latitude place the declination, and mark each with its proper name, N. or S.

(*a*) When the names are *alike*, take the *difference.*
(*b*) When the names are *unlike*, take the *sum.*

Under the difference or sum, put the zenith distance, and take the *sum* and *difference.*

Then add together, the log. *secant* of the latitude.
log. *secant* of the declination.
½ log. *havers.* of the sum.
½ log. *havers.* of the difference.

Reject the tens from the index of the sum of these four logs., and the remaining log. will be a *haversine.*

8. Find the hours, minutes, and seconds corresponding to this haversine.

(*a*) Taking them from the *top* of the page when the sun was observed *west* of meridian (or P.M. time at ship).

(*b*) And from the *bottom* of the table when the sun was observed *east* of meridian (or A.M. time at ship).

This will be the *apparent* time at ship.

9. To the apparent time at ship apply the equation of time, adding it, or subtracting it, according to the direction given in page I. of the month of the Nautical Almanac.

10. The result will be the *mean time at ship*, which must be marked with the proper *astronomical* day of the month.

11. Under the mean time at ship, place the mean time at Greenwich by *chronometer.* If the days of the month be different, add 24 hours to the one more advanced, and put the day *one back.* Take the *difference* of the times, which will give the longitude in *time.*

12. Turn this time into *degrees*, minutes, and seconds, which will give the *longitude* of the ship.

(*a*) West, when the Greenwich time is *greater* than the ship time.

(*b*) East, when the Greenwich time is *less* than the ship time.

Note.—To ascertain the *daily rate* when the errors on two different days are given, *see* (Additional Examples) at end of Book.

* When the altitude has been taken with an *artificial* horizon, the index error must be applied to it with its proper sign + or −, and the *result* divided by 2. Correct this for refraction and semidiameter as in ordinary cases, but not for *dip.*

EXAMPLE.

On September 30th, at about 8h. 20m. A.M., mean time, in lat. 30° 10′ N., and long., by account, 40° 0′ W., the obs. alt. of the sun's L.L. was 29° 51′ 50″, index error + 2′ 10″, height of the eye 18 feet, when a chron. showed 9h. 44m. 44s.; find the true longitude of the ship.

At G. M. noon, September 20th, the chron. was slow 1h. 15m. 0s., and its daily rate was 4·5s. losing.

(1) Approx. Green. date.

	h.	m.		
Time at ship	8	20	A.M.	Sep. 30th
	12			
Ast. T. ship	20	20		Sept. 29th
Long. in time	2	40		W.
Green. date	23	0		Sept. 29th

Note.—This time is not to be used to correct the declin. or eq. time.

(2) Green. Mean Time by Chron.

	h.	m.	s.	
Chron. showed	9	44	44	Sept. 30th
Error	1	15	0	slow
	10	59	44	
Rate 4·5 s. / Days 10			+0 45	losing
45·0				
	11	0	29	
Add	12			
Green. M. T.	23	0	29	Sept. 29th.

(4) Declin. p. II., N.A.

Declin. 29th	2°	25′	26″	S.
Corr. for 23h.		+22	23	
Declin.	2	47	49	S.

(4) Eq. of Time, p. II., N.A.

	m.	s.	
Eq. time, 29th	9	40·85	
Corr. for 23h.		+18·86	
Eq. time	9	59·71	

Sub. from App. Time.

(5)

(4) Change, p. I., N.A.

Hourly diff.	58″·41	
× by G.M.T.	23	
	17523	
	11682	
	6,0)134,343	
Corr. for 23h.	22′ 23″	

(4) Change, p. I., N.A.

Hourly diff.	·820
× by G.M.T.	23
	2460
	1640
Corr. for 23h.	18·860

(6) *To find the Zen. Dist.*

Obs. alt.	29°	51′	50″
In. error		+2	10
	29	54	0
Dip		−4	11
App. alt.	29	49	49
Ref.		−1	33
Tr. alt.	29	48	16
Semid.		+16	0
Tr. alt. centre	30	4	16
	90		
Zen. dist.	59	55	44

(7) *To compute the App. Time at Ship.*

Lat.	30°	10′	0″	N.	Sec.	0·063201
Declin.	2	47	49	S.	Sec.	0·000517
(7b) Sum	32	57	49			
Zen. dist.	59	55	44			
Sum	92	53	33	½ Hav.		4·860172
Diff.	26	57	55	½ Hav.		4·367658
				Haversine		9·291548

	h.	m.	s.	
(8b) App. time at ship	20	29	58	
Eq. time		−10	0	
(10) Ast. mean time ship	20	19	58	Sept. 29th
(3p) Green. mean time	23	0	29	Sept. 29th
(11) Long. in time	2	40	31	

Longitude 40° 7′ 45″ W. (12a.)

LONGITUDE BY CHRONOMETER. (MOON, PLANET, OR STAR.)

RULE.

1. HAVING found the Greenwich mean time by chronometer, as directed in Rule for Chronometer, page 76.

2. Take from the N. A. the R. Asc. mean sun, and correct it; also the Declin. and R. Asc. of the body, and correct them for Green. M. T. by chron. when necessary.

3. From the observed altitude get the true zenith distance.

To compute the hour angle.

4. Proceed as in (7) of Rule for Chron. page 77.

To find the mean time at ship.

5. To the hour angle add the R.A. of the body, and from the sum (increased if necessary by 24h.) subtract the R.A. mean sun. The remainder, diminished by 24h. if greater than 24h., will be the *mean time at ship*, which mark with the proper *astronomical* day of the month.

Then with the Green. mean time by chron. find the longitude, as in (11) and (12) of Rule, page 77.

EXAMPLE. (STAR).

Sept. 10th, at about 11h. 0m. P.M. mean time, in lat. 32° 18′ N., and long. D.R. 41° 22′ W., the obs. alt. of *a Lyræ* (west of mer.) was 44° 7′, index error + 1′ 50″, height of the eye 20 feet, when a chron. showed 1h. 5m. 20s.; required the longitude of the ship.

On Sept. 1st, at G.M. noon, the chron. was 38m. 14s. slow, and its daily rate was 1·8 sec. losing.

(1) G.M.T. by chron.	h. m. s. 13 43 51	Sept. 10th.	R. A. mean sun	h. m. s. 11 19 25	} (2)		
(3) Zen. dist.	45° 56′ 0″		R. A. *a Lyræ*	18 32 23			
			Dec. *a Lyræ*	38° 39′ 39″ N.			
(4) Star's hour angle	h. 3	m. 47	s. 6				
Star's R. Asc.	18	32	23				
Sum	22	19	29				
R.A. mean sun	11	19	25				
(5) Mean time ship	11	0	4	Sept. 10th.			
G.M.T. chron.	13	43	51	Sept. 10th.			
Long. in time	2	43	47				

Longitude 40° 56′ 45″ W.

LATITUDE BY DOUBLE ALTITUDE (SUN).

RULE.

1. GET a Greenwich date corresponding to *each* mean time at *ship*.

2. Take out the sun's declination from page II. of the month of the Nautical Almanac, and correct it separately for *each* Greenwich date. Take out also the sun's semidiameter.

3. Then with *each* declination find the corresponding polar distance.

4. Find the *polar angle in time.*

(*a*) When both times by chronometer are A.M. or both P.M., take their difference, which will give the polar angle *in time.*

(*b*) When one time by chronometer is A.M., and the other P.M., add 12 hours to the P.M. time, and take the A.M. time from it, the difference will be the polar angle *in time.*

Note.—The time at each observation should be taken by a *good* watch or chronometer, from which the *interval* between the two observations (polar angle) is to be obtained.

This should also be corrected for the *rate* of the watch or chronometer (if any of consequence) in the interval, adding it when *losing*, and subtracting it if *gaining*, which will give the polar angle in time.

5. Correct each observed altitude for index error, dip, semid., and (refraction — par.) then get the true altitudes.

6. Correct the *first* true altitude for the *run of the ship*, after which get the zenith distance. Also get zenith distance from second true altitude.

7. The first true altitude is corrected for *run*, as follows :—

Enter the Traverse Table (x. Inman) with the *distance* run in the *interval* in the *dist.* column, and the angle included between the true bearing of the sun at the *first* observation, and the true course of the ship (in the *interval*) as a *course*, and take out the corresponding *diff. lat.* in the same manner as when working the day's work.

(*a*) When the angle found above is *less* than 8 points, *add* the diff. lat. taken out of the Traverse Table, to the *first* true altitude.

(*b*) When the angle found above is *more* than 8 points, enter the Traverse Table with what it wants of 16 points, and *subtract* the diff. lat. from the *first* true altitude.

(*c*) When the angle found above is 0 points, *add the whole distance run in the interval* to the *first* true altitude.

(*d*) When the angle found above is exactly 16 points, *subtract the whole distance run in the interval* from the *first* true altitude.

(*e*) When the angle found above is exactly 8 points, there will be no correction.

Note 1.—The *tenths* of miles found in the Traverse Table can be turned into *seconds*, by multiplying them by 6.

Note 2.—By the *greater* bearing is meant the bearing of the body at the moment of observing the *lesser* altitude of the two; or its bearing when farthest from the meridian (Noon).

Note 3.—By the *less* bearing is meant the bearing of the body at the moment of observing the *greater* altitude of the two; or its bearing when nearest to the meridian (Noon).

Note 4.—When the bearings lie on opposite sides of the meridian, and are equal (that is, the same distance from N. or from S.), either may be taken as the less bearing.

To Compute the Latitude.

To find *Arc* (1).

Add together

the log. sin. of the pol. dist. at G.B.
log. sin. of the pol. dist. at L.B.
log. havers. of the polar angle (in time),

reject the tens in the index of the sum; with this result as a log. *haversine*, take out the arc in degrees, and mark it (*a*), which will be arc (1) when the difference of the polar distances is nothing, or *very small*. (See *Note*.)

Note.—When the diff. polar distances exceed 40′, to the nat. versine (Tab. v. Inman) of arc (*a*) add the nat. versine of diff. pol. dists. With the sum as a nat. versine take out the arc and mark it (1).

To find *Arc* (2).

Under arc (1) place the pol. dist. at *greater* bearing, and take

their difference, under which diff. place the pol. dist. at *less* bearing, take the sum and diff.

Add together

the log. cosec. of arc. (1),
log. cosec. of P.D.G.B.,
$\frac{1}{2}$ log. havers. of the sum,
$\frac{1}{2}$ log. havers. of the diff.,

reject the tens in the index of the sum of these four logs. and take out the result as a *haversine*, in degrees. Call this arc (2).

To find *Arc* (3).

Again, under arc (1) place the zen. dist. at *greater* bearing, and take their difference, under which difference place the zen. dist. at *less* bearing, take the sum and diff.

Add together

the log. cosec. of arc (1),
log. cosec. of Z.D.G.B.,
$\frac{1}{2}$ log. havers. of the sum,
$\frac{1}{2}$ log. havers. of the diff.,

reject the tens in the index of the sum of these four logs. and take out the result as a log. *haversine*, in degrees. Call this arc (3).

To find *Arc* (4).

Take the *difference* between arcs (2) and (3), for arc (4), unless the distance arc joining the places of the sun at the two observations passes *between* the zenith and pole; in which case alone take the *sum* of arcs (2) and (3) for arc (4).

To find the *Latitude.*

Add together

the log. sin. of the P.D.G.B.,
log. sin. of the Z.D.G.B.,
log. havers. of arc (4),

reject the tens in the index of the sum, and take out the log. *haversine* of the remaining log. in degrees. Call this arc (5).

To the natural versine (Tab. v. Inman) of this arc (5), *add* the natural versine of the difference between the polar and zenith distances at the *greater* bearing. The sum will be natural versine of the co-latitude, which subtracted from 90° will give the latitude required.

Note.—When *near the equator* the arc may be greater than 90°, in which case subtract 90° from it, the remainder will then be the latitude of a *contrary* name to the latitude by *account.*

EXAMPLE.

September 24th, in lat. D.R. 10° 25′ N. and long. 168° 15′ E., the following observations were taken to find the latitude:—

M. T. Ship.	Chron. Times.	Obs. alt. Sun's L.L.	True Bearings.
11h. 15m. A.M.	oh. 5m. 15s.	75° 59′ 20″	S. 39° E.
3 10 P.M.	3 56 30	39 27 30	S. 82 W.

Index error +1′ 30″, height of the eye 18 feet; run of the ship in the interval S. 30° E., 29 miles.

(1) Green. date at Gr. Bg.	(4a) Polar ∠ (in Time)	(1) Green. date at Lr. Bg.
h. m.		h. m.
Time at ship 3 10 Sept. 24th	Chronometer Times.	Time at ship 11 15 A.M. Sept. 24th
24	h. m. s.	12
Ast. sh. T. 27 10 Sept. 23rd	0 5 15	Ast. sh. T. 23 15 Sept. 23rd
Long. in time 11 13 E.	3 56 30	Long. in time 11 13 E.
Gr. date 15 57 Sept. 23rd	3 51 15	Gr. date 12 2 Sept. 23rd
	Polar ∠ (in Time)*	

(2) Declin. at Gr. Bg.	Change.	(2) Declin. at Lr. Bg.	Change.
Dec. Sept. 23rd 0° 5′ 2″ S.	58″·48	Dec. Sept. 23rd 0° 5′ 2″ S.	58″·48
Corr. +15 36	16	Corr. +11 42	12
Declin. 0 20 38 S.	35088	Declin. 0 16 44 S.	6,0)70,1·76
90	5848	90	Corr. 11′ 42″
Pol. dist. (3) 90 20 38	6,0)93,5·68	Pol. dist. (3) 90 16 44	
	Corr. 15′ 36″		

* This is not necessarily the mean time at Greenwich, but the time shown by a *good* watch or chronometer, in order to obtain the polar angle as accurately as possible.

To find the Zenith Distances.

(5) *First obs. alt.* (*L.B.*)

Obs. alt.	79° 59′ 20″
Index error	+ 1 30
	76 0 50
Dip	− 4 11
	75 56 39
Ref.	− 0 12
	75 56 27
Semid.	+ 15 59
Tr. alt. centre	76 12 26
(7a) Angle between bearing of sun at first obs. (L.B.) and run of ship, 8 pts., dist. 29m.	+ 28 42
True alt.	71 41 8
	90
(6) Zen. dist.	13 18 52

(5) *Second obs. alt.* (*G.B.*)

Obs. alt.	39° 27′ 30″
Index error	+ 1 30
	39 29 0
Dip	− 4 11
	39 24 49
Ref.	− 1 4
	39 23 45
Semid.	+ 15 59
True alt. centre	39 39 44
	90
Zen. dist.	50 20 16

To find Arc (1).

P.D.G.B.	90° 20′ 38″	Sin.	9·999992
P.D.L.B.	90 16 44	Sin.	9·999995
Pol. angle	3h. 51m. 15s.	Hav.	9·368574
		Havers. arc (*a*)	9·368561

Arc (1) 57° 48′ 45″.

To find Arc (2).

Arc (1)	57° 48′ 45″	Cosec.	0·072471
P.D.G.B.	90 20 38	Cosec.	0·000008
Diff.	32 31 53		
P.D.L.B.	90 16 44		
Sum	122 48 37	½ Hav.	4·943503
Diff.	57 44 51	½ Hav.	4·683929
		Havers.	9·699811

Arc (2) 90° 6′ 45″.

To find Arc (3)

Arc (1)	57° 48′ 45″	Cosec.	0·072471
Z.D.G.B.	50 20 16	Cosec.	0·113612
Diff.	7 28 29		
Z.D.L.B.	13 18 52		
Sum	20 47 21	½ Hav.	4·256265
Diff.	5 50 23	½ Hav.	3·707196
		Havers.	8·149544

Arc (3) 13° 38′ 30″.

To find the Latitude.

Arc (2)	90° 6′ 45″		
Arc (3)	13 38 30		
Arc (4)	76 28 15	Hav.	9·583233
P.D.G.B.	90 20 38	Sin.	9·999992
Z.D.G.B.	50 20 16	Sin.	9·886388
Diff.	40 0 22	Hav.	9·469613

Arc (5) 65° 46′ 45″.

Parts for 45″	199
Vers. Arc (5)	·0589546
Vers. diff. P. and Z.D.	·0233956
Parts for 22″	69
Versin. (sum)	·0823770
Arc	79° 51′ 0″ (co-latitude)
	90
Latitude	10 9 0 N.

DOUBLE ALTITUDE,

By two altitudes of different bodies observed at the same time.

(TWO STARS.)

RULE.

1. TAKE from the N. A. the declinations of the two bodies, and find the polar distances, as in (*a*) or (*b*) of 3, Rule for Double Altitude, and take their difference.

2. Also take from the N.A. the right ascensions of the two bodies, their difference (or if greater than 12h. what it wants of 24h.) will be the *polar angle* in time.

3. Correct the two observed altitudes, and get the zenith distances.

TO COMPUTE THE LATITUDE.

4. To find Arc (1).

Add together

the log. sin. of the pol. dist. at G.B.,
log. sin. of the pol. dist. at L.B.,
log. havers. of the polar angle (in time),

reject the tens in the index of the sum ; with this result as a log. haversine, take out the arc in degrees, and mark it (*a*). To the nat. versin. of (*a*) add the nat. versin. of the diff. pol. dist. With the sum as a nat. versin. take out the arc and mark it (1).

The remainder of the rule is the same as for the sun.

Note.—In finding arc (4), it will sometimes happen that the sum of arcs (2) and (3) has to be taken. This will be the case only when the distance arc joining the places of the two bodies passes *between* the zenith and pole. If the sum of (2) and (3) be greater than 180°, the supplement to 360° must be considered as the sum.

Arcs (1) and (2) may be taken by inspection from Rear-Admiral Shadwell's Star Tables, which afford a very easy and expeditious method of finding them.

EXAMPLE.

Sept. 30th, in lat. by account, 41° 15′ S., the following altitudes were observed at the same time.

Obs. Alt. α Hydræ (G.B.)	*Obs. Alt. α^2 Libræ (L.B.)*
46° 22′ 40″	41° 5′ 20″
Index error −1 16	−4 16

The height of the eye was 24 feet; required the latitude.

Declin. α Hydræ.	*Declin. α^2 Libræ.*
8° 5′ 6″ S.	15° 29′ 3″
90	90
P.D. 81 54 54	P.D. 74 30 57

	h.	m.	s.			
R. Ascen. α Hydræ	9	21	2	Z.D. α Hydræ	43°	44′ 20″
R. Ascen. α^2 Libræ	14	43	20	Z.D. α^2 Libræ	49	4 52
Polar angle	5	22	28			

To find Arc (1).

P.D.G.B. 81° 54′ 54″	Sin. 9·995663	P.D.G.B.	81° 54′ 54″
P.D.L.B. 74 30 57	Sin. 9·983945	P.D.L.B.	74 30 57
Pol. angle 5h. 22m. 28s.	Hav. 9·621690	Diff.	7 23 57
Log. havers.	9·601298		
Arc (*a*) 78° 22′ 45″			
Nat. vers. (*a*)	·0798565	Arc (2) = 75° 42′ 30″	
Nat. vers. diff. P.D.	·0008326	Arc (3) = 40 32 48	
Nat. vers.	·0806891	Arc (4) = 35 9 42	
Arc (1)	78° 51′ 57″		

Arcs (2), (3), and (4), and the latitude are found as in Rule for Double Altitude.

Latitude 41° 23′ 30″ S.

TO FIND THE TIME OF THE MOON'S PASSAGE OVER A GIVEN MERIDIAN.

RULES.

In **West** *Longitude.*

(A) TAKE from the Nautical Almanac (page IV. of the month) the moon's meridian passage on the given day; underneath which place the moon's passage on the *following* day, and take their difference. (Diff. moon's mer. pass.)

Then enter Table k. (Inman) with the diff. moon's mer. pass on the left hand of the page to *nearest minute*, and longitude of ship to *nearest degree* at the top, and take out the *correction*, which is to be added to the *first* passage; the sum will be the time of moon's passage at *ship*, of the day of first passage.

In **East** *Longitude.*

(B) Take from the Nautical Almanac (page IV. of the month) the moon's meridian passage for the given day; underneath which place the moon's passage on the *preceding* day, and take their difference. (Diff. moon's mer. pass.)

Then enter Table k. (Inman) with this diff. moon's mer. pass. on the left hand of the page to *nearest minute*, and longitude of ship to *nearest degree* at the top, and take out the *correction*, which *subtract* from the *first* passage; the remainder will be the time of moon's passage at *ship*, of the day of first passage.

Note 1.—In *west* longitude, should the sum of the moon's mer. pass. (from N.A.) and the *correction* from Table k. (together) exceed 24h., reject 24h., and put the *date* one day forward.

Note 2.—In *east* longitude, should the *correction* from Table k. be more than the time of the moon's mer. pass. (from N.A.), add 24h. to the passage before subtracting the correction, and put the *date* one day back.

EXAMPLE (A).—Longitude *West.*

On Sept. 19th, in long. 60° W., find the moon's meridian passage at ship.

Moon's passage at Gr. 19th Sept. (given day)		7h.	58m.	(p. IV. N.A.)
Ditto	20th Sept. (day following)	8	48	
Diff. moon's mer. pass.		0	50	
Enter Tab. k. with diff. pass. 50m. (taken as 48m.) at side, and long. 60° at top, gives *correction*		+	8	*added* to first mer. pass.
Moon's meridian passage at *ship*		8	6	Sept. 19th

EXAMPLE (B).—Longitude *East.*

Find the moon's mer. pass. at a ship. in long. 79° E., on Sept. 20th.

Moon's passage at Gr. 20th Sept. (given day)		8h.	48m.	(p. IV. N.A.)
Ditto	19th Sept. (preceding day)	7	58	
Diff. moon's mer. pass.		0	50	
Enter Tab. k. with diff. pass. 50m. (taken as 48m.) at side, and long. 79° (taken as 80°) at top, which gives the *correction*		−	11	subtracted from first mer. pass.
Moon's mer. pass. at *ship*		8	37	Sept. 20th

TO CORRECT THE MOON'S SEMIDIAMETER AND HORIZONTAL PARALLAX.

RULES.

1. Having found the Greenwich date, take out the moon's semi-diameter from page III. of the Nautical Almanac, as follows:—

(A) When the Greenwich date is *less* than 12h., take out the semid. for *noon* of the day of the Greenwich date; underneath which place the semid. for *midnight* of the *same* day, and take their difference.

(B) When the Greenwich date is *more* than 12h., take out the semid. for *midnight* of the day of the Greenwich date; underneath which place the semid. for *noon* of the *following* day, and take their difference.

2. Then multiply this difference by the *hours* of the Greenwich date, and divide the product by 12; the quotient will be the *correction.*

Note.—If the G.D. exceeds 12h., use what it is *above* 12 hours.

3. If the semid. is *increasing*, add the correction to the semid. *first* taken out, the sum will be the semidiameter required.

4. If the semid. is *decreasing*, subtract the correction from the semid. *first* taken out; the remainder will be the semidiameter required.

Note.—The above rules are applied in precisely the same manner when correcting the *horizontal parallax.*

EXAMPLE (I).

Find the moon's semidiameter and horizontal parallax on Sept. 16th, at 9h. M.T.G.

Semid.		Change.	Hor. Par.		Change.
	′ ″	″		′ ″	″
Semid. *noon* 16th Sept.	14 49·5	2·1	Hor. Par. *noon* 16th Sept.	54 18·7	7·7
Ditto *mid.* 16th Sept.	14 51·6	× by 9	Ditto *mid.* 16th Sept.	54 26·4	× by 9
Difference in 12h.	2·1	12) 18·9	Difference in 12h.	7·7	12) 69·3
Add to semid. at *noon*	+ 1·6		Add to H.P. at *noon*	+ 5·8	
		Corr. 1″·6			Corr. 5″·8
Semidiameter	14 51·1	nearly	Hor. Par.	54 24·5	nearly

EXAMPLE (II).

Green. date *more* than 12 hours.

Find the moon's semidiameter and horizontal parallax on Sept. 19th, at 15h. M.T.G.

Semid.		Change.	Hor. Par.		Change.
	′ ″	″		′ ″	″
Semid. *mid*, 19th Sept.	15 18·0	6·2	Hor. Par. *mid.* 19th Sept.	56 3·3	22·9
Semid. *noon* 20th Sept.	15 24·2	× by 3*	Hor. Par. *noon* 20th Sept.	55 26·2	× by 3*
Diff. in 12h.	6·2	12) 18·6	Difference in 12h.	22·9	12) 68·7
Add to semid. at *mid.*	1·5		Add. to H.P. at *mid.*	5·7	
		Corr. 1″·5			Corr. 5″·7
Semidiameter	15 19·5	nearly	Hor. Par.	55 9·0	nearly

* Here, the diff. in 12h. is multiplied by 3, or what 15h. (the G.D.) is over 12h. (see B).

* Here, the diff. in 12h. is multiplied by 3 (see B).

TO CORRECT THE MOON'S DECLINATION.

RULE.

1. Find the time of the moon's passage at ship (*see* Rules, p. 87), and with the longitude in time, get the Greenwich date.

2. Take out the moon's declination from the N.A. for the *hours* of the Greenwich date; also take out from the adjoining column the diff. declin. for 10 *minutes*.

3. Multiply the diff. declin. by the *minutes* of the Greenwich date, divide the product by 10 and then by 60; the result will be the *correction* in minutes and seconds.

Or, remove the decimal point in the diff. dec. one place to the *left*, which will give the diff. dec. for *one* minute; then multiply by the *minutes* of the Greenwich date, and divide the product by 60; the result will be the *correction* in minutes and seconds.

4. When the declin. is *increasing*, add the correction, which will give the declin. required.

5. When the declin. is *decreasing*, subtract the correction; the remainder will be the declin. required.

Note 1.—When the correction is *subtractive* and is more than the declin. taken out, the difference between the correction and declination will give the declin. required, but which is to be marked with a *contrary* name to the declin. taken out.

EXAMPLE I.

Find the moon's declin. on Sept. 15th, at 5h. 35m. M.T.G.

(2) Declin.		(3) Diff. dec. in 10m.	
Declin. Mar. 8th, at 5h.	16° 47′ 5″S.	Diff. dec. in 10m.	36″·44
Corr. for 35m.	+ 2 7	× by minutes of Green. date	35
			18280
			10932
Declination	16 49 12 S.		6,0)127,5·40
			Corr. 2′ 7″

EXAMPLE II.

Find the moon's declin. on Sept. 23rd, at 22h. 52m. M.T.G.

(2) Declin.		(3) Diff. dec. in 10m.		
Declin. Sept. 23rd, at 22h.	0° 7′ 38″S.	Diff. dec. in 10m.	111″·84	decimal point removed one place to left.
Corr. for 52m. (*Note* 1)	0 9 42		11·184	
		× by minutes of Gr. date.	52	
Declination	0 2 4 N.		22368	
			55920	
			6,0)58,1·568	
			Corr. 9′ 42″	

MOON'S RIGHT ASCENSION.

To correct the moon's right ascension, take out the R.A. for the *hour* of the Green. date, and find the increase of R.A. between that hour and the next following. Then ascertain the proportion of increase for the *minutes* of the G.D., and *add* it to the R.A. taken out, which will be the R.A. required.

TO FIND THE LATITUDE BY THE MERIDIAN ALTITUDE OF THE MOON.

RULE.

1. Find the time of the moon's meridian passage at *ship* (by Rule, page 87), and with the given longitude get the Greenwich date.

2. For this date take from the Nautical Almanac the moon's declination, moon's semidiameter, and horizontal parallax, and correct them.

3. Then correct the observed altitude for index error, dip, and (augmented) semid.,* and get the apparent altitude.

4. Enter Table w. (Inman) with the degrees and minutes of *apparent altitude* at the side, and minutes of moon's hor. par. at the top, and take out the correction from the column marked $\left|\begin{smallmatrix}\text{corr.}\\+\end{smallmatrix}\right|$ and place it under the apparent altitude.

Note.—If the horizontal parallax is *less* than 54′ (the limit of Table w.), take out the correction for 54′, and also the parts for the seconds which the hor. par. is *less* than 54′; then *subtract* these parts from the "corr." given in the table, the remainder will be the correction required.

5. Then from the small table at the right-hand side of the same page, with the *seconds* of hor. par. in the column marked (″), take out the seconds given in the adjoining column marked corr., which place under the corr. obtained in (4).

Add both these quantities to the app. alt., the sum will be the true altitude.

6. Subtract the true alt. from 90° for the zenith distance (Z.D.).

Mark the Z.D. N. or S., according as the observer is N. or S. of the moon.

Under the Z.D. place the declin. also marked N. or S.

7. When the Z.D. and declin. are of the *same* name, their sum will be the latitude, of the name of either.

8. When the Z.D. and declin. are of contrary names, take the less from the greater; the difference will be the latitude, of the name of the *greater*.

* The *augmentation* of the moon's semid. is taken out from Table *g*, Inman, with alt. at side, and semid. at top.

Ex. to Note.—Suppose the app. alt. to be 30° 20′, and the hor. par. 53′ 55″. The "corr." corresponding to 54′ of hor. par. and app. alt. 30° 20′ is 44′ 58″ and the parts (in the small table) for the 5″, by which the hor. par. is *less* than 54′ are 4, these *subtracted* from 44′ 58″ give 44′ 54″, the correction required.

EXAMPLE.

September 15th, in long. 27° W., the obs. mer. alt. of moon's L.L. was 54° 15′ 30″ (Z.N.), index error − 3′ 30″, height of eye 19 feet; required the latitude.

(1) Gr. Date from pass. at Ship.	h. m.
Moon's pass. Sept. 15th	4 45
Ditto, Sept. 16th	5 32
Diff. pass.	0 47
Enter Tab. k. with diff. pass. 48, and long. 25°, gives	+ 3
Moon's pass. at ship	4 48
Long. in time	1 48W.
Gr. date, Sept. 15th	6 36

(2) Declination.		
Declin., 15th, at 6h.	16° 50′ 42″ S.	
Corr. for 36m.	2 8	
Declin.	16 52 50 S.	

Change in 10m.	
	35″·68
× by min. of G.D.	36
	21408
	10704
	6,0)12,8,4·48
Corr. for 36m.	2′ 8″

(2) Semid. p. III., N.A.	′ ″
Semid. noon, 15th	14 47·3
Ditto, mid., 15th	14 48·0
Diff. 12h.	·7
Corr. for 6h. 36m.	+ ·4
Semid.	14 47·7
Augmentation alt. 54°	+ 11·3
A.S.D.	14 59·0

Change.	
	·7
× by hours of G.D.	6·5
	12)4·55
Corr.	0·4

(2) Hor. par. p. III., N.A.	′ ″
Hor. par. noon, 15th	54 11·0
Ditto, mid., 15th	54 13·5
Diff. in 12h.	2·5
Corr. for 6h. 36m.	+ 1·4
Hor. par.	54 12·4

Change.	
	2″·5
× by min. of G.D.	6·5
	125
	150
	12)16·25
Corr.	1″·4

To find the Latitude.

(3)	Obs. mer. alt.	54° 15′ 30″
	Index error	− 3 30
		54 12 0
	Dip	− 4 17
		54 7 43
	Aug. S. D.	+ 14 59
	App. alt. centre	54 22 42
(4)	With app. alt. 54° 22′ at side, and hor. par. 54 at top, gives	+ 30 46
(5)	Small table, with 59″ of hor. par. under Corr. gives	+ 7
	True alt.	54 53 35
		90
(6)	Zen. dist.	35 6 25 N.
	Declin.	16 52 50 S.
(8)	Latitude	18 13 35 N.

LONGITUDE BY LUNAR. (SUN.)

RULE.

1. With the mean time at ship and longitude get a Greenwich date.

2. For this date take out of the Nautical Almanac (page II. of the month) the sun's declin., the equation of time, and the sun's semid.

Also, from page III. of the month, take out the moon's semid. and horizontal parallax.

3. Correct the sun's declin. and the equation of time.

4. Correct the moon's semid. and hor. parallax.

5. Correct the *sun's* observed altitude

$$\text{for} \left\{ \begin{array}{c} \text{Index error,} \\ \text{dip,} \\ \text{semid.} \end{array} \right\} \text{and get the app. alt. (A.A.)}$$

then apply the refraction (Tab. m.), and find the true alt.: also get the zenith dist. (Z.D.).

6. Correct the *moon's* observed altitude

$$\text{for} \left\{ \begin{array}{c} \text{Index error,} \\ \text{dip,} \\ \textit{augmented} \text{ semid.} \end{array} \right\} \text{and get the app. alt. (A.A.)}$$

then apply the correction in alt. (Tab. w.), and find the true alt.: also get the zenith dist. (Z.D.)

7. To take out the auxiliary angle A. (Tab. w.)

Whilst Tab. w. is being used for the corr. in alt. look in the *adjoining* column to the *right* of that marked $\left| \begin{smallmatrix} \text{corr.} \\ + \end{smallmatrix} \right|$, and under $\left| \begin{smallmatrix} \text{A} \\ 60^{\circ} \end{smallmatrix} \right|$ in the *same* line with the "corr." just taken out, will be found the minutes and seconds of the auxiliary angle A.

Then, in the small table on the right-hand side of the page, and with the *seconds* of hor. par. take out the seconds in the column under | A | .

Next, go to the small table at the bottom of the page, and with the *nearest* degree to the app. alt. of the *sun*, take out the seconds given in the column under | ⊙ | .

Add these three quantities to 60°, which will be the auxiliary angle A required.

8. Correct the observed distance for index error (if any). To this result *add* the sun's semid. and also the moon's *augmented* semid. ; the sum will be the apparent distance (A.D.).

9. To compute the **true distance.**

Add together the two true zenith distances (Z.D.), and mark the sum *vers.*

Add together the two apparent altitudes (A.A.), under this sum put the auxiliary angle A., take the sum and difference, and mark each *vers.*

Under the app. dist. (A.D.) put the auxiliary angle (aux. ∠), take the sum and difference, and mark each *vers.*

10. From Table v. (Inman), take out the nat. versine of the degrees and minutes of each of the above arcs (marked *vers.*), using, however, only the right-hand *five* figures of each versine : at the same time take from the opposite page of the table the parts for *seconds* of each arc.

Note.—If any arc marked *vers.* be greater than 180°, take the vers. of what it wants of 360°.

11. Place these versines in one column, and their parts in another. Add up each column separately, and then make one sum of both columns, rejecting the tens in the fifth place to the *left.*

12. Then look in the column of versines marked with the *degree* of the *apparent* distance, or in the column preceding or following that degree, when the five figures in the above sum, or the next *less* to them, will give the degrees and minutes of the *true* distance (T.D.).

13. If there is any difference between the figures of the versine found from the table and those sought for, the seconds corresponding to this difference must be taken from the opposite page ("Parts for Seconds"), and added to the degrees and minutes of the true distance.

To compute the mean time at Greenwich.

14. Find in the Nautical Almanac, on the *given* day, two distances of the sun and moon *between* which the true distance lies—that is, two distances, one of which is *greater* and the other *less* than the true distance just found.

Under the true distance (T.D.), place that distance which comes *first* in order in the Nautical Almanac, which mark with the *hour* denoted at the head of its column; and at the same time take out the proportional logarithm | P.L. of diff. | given in the *adjoining* column on the right.

Take the diff. between the T.D. and the one given in the Nautical Almanac, and find its corresponding proportional logarithm from Table r. (Inman).

15. Then take the less prop. log. from the greater: the result will be the prop. log. of a portion of time, to be added to the *hour* denoted at the head of the column of distances in the N.A.; the sum will be the *mean time at Greenwich* (expressed astronomically), which mark with the day given in the Nautical Almanac.

Note.—If great accuracy be required, such as when fixing the longitude of a place on shore, the moon's hor. par. and the latitude of the place should be corrected for the spheroidal figure of the Earth, by Table h. (Inman).

16. To compute the **mean time at ship.**

This is done precisely as in working a *sun* chronometer. (See 7 of that Rule.)

Note.—The sun is here the body from which the ship time is deduced.

Take the difference between the Greenwich mean time (obtained as above) and the ship mean time (reducing them, if necessary, to the *same* date by adding 24h. to the least of the two, and putting it back a day).

17. This difference will be the longitude in time, which, when turned into degrees, &c., will give the longitude of the ship.

18. (*a*) *East*, when the Greenwich time is the *less* of the two.
(*b*) *West*, when the Greenwich time is the *greater* of the two.

EXAMPLE.

September 15th, at 3h. 20m. P.M. mean time, in lat. 35° N. and long. D.R. 25° 30′ W., the following lunar was taken :—

	Obs. alt. sun's L.L.	Obs. alt. moon's L.L.	Obs. dist.
	32° 39′ 50″	33° 25′ 20″	74° 11′ 10″
Index error	— 1 25	+ 1 40	+ 1 5

Height of eye 24 feet; required the longitude.

(1) Green. date.		(3) Sun's declin. p. II., N.A.		(3) Change.	
	h. m.		° ′ ″		″
Time at ship	3 20 Sept. 15th	Declin., 15th	3 1 19 N.		57·81
Long. in time	1 42 W.	Corr.	−4 49	× G.D.	5
Gr. date	5 2 Sept. 15th	Declin.	2 56 30 N.		6,0)28,9·05
				Corr.	4′ 49″

Sun's semid.		(3) Eq. time, p. II., N.A.		Change.	
	′ ″		m. s.		s.
Semid.	15 57	Eq. time, 15th	4 50·0		·879
		Corr.	+4·39	× G.D.	5
		Eq. time	4 54·39	Corr.	4·395
		Sub. from app. time.			

(4) Moon's semid. p. III., N.A.		Change.	(4) Hor. par. p. III., N.A.		Change.
	′ ″	″		′ ″	″
Semid. noon, 15th	14 47·3	·7	Hor. par. noon, 15th	54 11	2·5
Ditto, mid., 15th	14 48·0	× 5	Ditto, mid., 15th	54 13·5	× 5
Diff. in 12h.	·7	12)3,5	Diff. in 12h.	2·5	12)12·5
Corr. for 5h.	+·3	Corr. ·3	Corr. for 5h.	+ 1·0	Corr. 1″·0
Semid.	14 47·6		Hor. par.	54 12	
Augmentation, Tab. g. alt. 32°	+ 7·5				
Aug. semid.	14 55·1				

(3) To find Sun's Z.D.

Obs. alt.	32° 39′ 50″
Ind. error	− 1 25
	32 38 25
Dip	− 4 49
	32 33 36
Semid.	+ 15 57
A.A.	32 49 33
Ref.	− 1 23
True alt.	32 48 10
	90
Zen. dist.	57 11 50

(6) To find Moon's Z.D.

Obs. alt.	33° 25′ 20″
Ind. error	+ 1 40
	33 27 0
Dip	− 4 49
	33 22 11
Aug. S.D.	+ 14 55
A.A.	33 37 6
Corr. Tab. w.	+ 42 32
	+ 0 10
True alt.	34 19 48
	90
Zen. dist.	55 40 12

(8) To find App. Dist.

Obs. dist.	74° 11′ 10″
Ind. error	+ 1 5
	74 12 15
⊙ Semid.	+ 15 57
☽'s A.S.D.	+ 14 55
A.D.	74 43 7

(7) Auxiliary angle A.

	60° 0′ 0″
Corr. Tab. w.	+ 16 17
Seconds of par.	+ 0 4
	+ 0 3
Aug. angle	60 16 24

(9) To compute the True Distance.

				(10)		(11)
Sun's Z.D.	57°	11′	50″			(Parts for seconds.)
Moon's Z.D.	55	40	12			
Sum	112	52	2	Vers.	88588 ...	13
Sun's A.A.	32°	49′	33″			
Moon's A.A.	33	37	6			
Sum	66	26	39			
Aux. angle	60	16	24			
Sum	126	43	3	Vers.	97858 ...	12
Diff.	6	10	15	Vers.	05786 ...	8
A. dist.	74°	43′	7″			
Aux. angle	60	16	24			
Sum	134	59	31	Vers.	06901 ...	107
Diff.	14	26	43	Vers.	31562 ...	50
					30695	190
				Parts	190	
				Vers.	30885	
(12)					30880 gives	74° 23′ 0″
					85 parts give	18 (see 13)
					True distance	74 23 18

(14) To compute the Mean Time at Greenwich.

True dist.	74°	23′	18″		
Dist. at III. hours from Naut. Almc.	73	26	55	P.L.	·34480
Diff.	0	56	23	P.L.	·50412
(Table r. Inman)				P.L. Diff.	·15932

	h.	m.	s.	
	2	4	43	
Hour in Naut. Almc.	3	0	0	
M.T. Green.	5	4	43	Sept. 15th (see 15.)

(16) To compute the Mean Time at Ship.

Lat.	35°	0′	0″	N.	Sec. 0·086636
Dec.	2	56	30	N.	Sec. 9·000573
Diff.	32	3	30		
Z.D.	57	11	50		
Sum	89	15	20		½ Hav. 4·846640
Diff.	25	8	20		½ Hav. 4·337680
				Havers.	9·271529

	h.	m.	s.	
App. time at ship	3	24	54	
Eq. time		— 4	54	
Mean time at ship	3	20	0	Sept. 15th
Mean time at Green.	5	4	43	Sept. 15th

(17) Longitude in time 1 44 43

Longitude 26° 10′ 45″ W. (18. *b*.)

LONGITUDE BY LUNAR.

(STAR, OR PLANET, AND MOON.)

1. Get a Greenwich date as in rule for Sun Lunar.

2. For this date take from the N.A. the moon's semid. and hor. par., and correct them.

3. Take out the R.A. mean sun, and correct it.

4. Take out the R. Asc. and declin. of the body from whose altitude the mean time at ship is to be deduced, and, when necessary, correct them.

5. From the observed altitudes get the zenith distances.

Note.—When a planet is observed, take out its hor. par. from the N.A. and *add* the *parallax in altitude* (found in Tab. b., Inman) to the *app.* alt. after it has been corrected for refraction, and before finding the Z.D.

6. To the observed distance corrected for index error, *add* the moon's augmented semid. when the limb of the moon observed is *nearest* to the star (N.L.); but *subtract* it from the distance when the limb observed is *furthest* from the star (F.L.). The result will be the apparent distance (A.D.).

7. Take out the auxiliary angle from Tab. (w.) Inman, in the same manner as directed in (7) for Sun Lunar, p. 93; but in using the small table at the bottom, take out the corr. given in the column under | * | instead of that under | ⊙ | .

Note.—If a planet be observed, take the second corr. for A from Tab. (c.) Inman, instead of using the small table for | ⊙ | or | * | .

8. Compute the **true distance** by the Rule for Sun Lunar, see (9) to (13), p. 94.

9. **To compute the mean time at Greenwich.**

Find in the Nautical Almanac on the *given day* two distances of the moon and star, or moon and planet (as the case may be) *between* which the true distance lies, and proceed to find the *mean time at Greenwich*, as in Sun Lunar.

10. **The mean time at ship** is computed by the Rule for Chronometer. (Star, Planet, or Moon.)

11. The longitude of the ship is found by taking the difference between the Greenwich mean time and ship mean time, as in rule for Sun Lunar.

EXAMPLE.

(*Moon and Star.*)

Sept. 22nd, at 8h. 30m. P.M. mean time, in lat. 40° 10′ S., and long. by account 52° 30′ W., the following lunar observation was taken :—

Obs. alt. *Antares.*	Obs. alt. moon's L.L.	Obs. dist. N.L.
36° 6′ 30″ (W. of mer.)	45° 32′ 0″	91° 30′ 40″
Ind. error — 3′ 10″	+ 1 40	— 2 50

Height of the eye 22 feet; required the longitude.

	h. m.		
Time at ship	8 30	Sept. 22	
Long. in time	3 30	W.	
Green. date	12 0	Sept. 22	

	°	′	″
Zen. dist. *Antares*	54	2	37
Zen. dist. moon	43	34	56
True distance	91	10	23

Elements from N.A. corrected.

		′	″
Moon's aug. semid.		16	9
Moon's hor. par.		58	28
	h	m.	s.
R.A. mean sun	12	6	27
R.A. *Antares*	16	21	14
	°	′	″
Dec. *Antares*	26	7	53 S.
App. dist.	91	43	59
Aux. angle	60	23	11
	h.	m.	s.
Star's hour angle	4	15	8

	h.	m.	s.	
M.T. Green.	11	59	25	Sept. 22.
M.T. ship	8	29	55	Sept. 22.
	°	′	″	
Longitude	52	22	30	W.

NAUTICAL ASTRONOMY.

EXAMPLES.

TO FIND THE LATITUDE BY THE MERIDIAN ALTITUDE OF A STAR.

1. April 1st, the obs. mer. alt. of *Regulus* was 64° 10′ (Z.N.), index error −1′ 40″, height of the eye 14 feet; required the latitude.

Ans. Latitude 38° 31′ N.

2. May 10th, the obs. mer. alt. of *Spica* was 59° 15′ (Z.S.), index error +2′, height of the eye 20 feet; required the latitude.

Ans. Latitude 41° 16′ S.

3. June 12th, the obs. mer. alt. of *Arcturus* was 50° 10′ (Z.S.), index error −2′ 20″, height of the eye 18 feet; required the latitude.

Ans. Latitude 20° 4′ S.

4. June 20th, the obs. mer. alt. of *Aldebaran* was 70° 5′ (Z.S.), index error +1′ 10″, height of the eye 24 feet; required the latitude.

Ans. Latitude 3° 45′ S.

5. July 15th, the obs. mer. alt. of *Castor* was 51° 25′ (Z.S.), index error −2′ 40″, height of the eye 17 feet; required the latitude.

Ans. Latitude 6° 32′ S.

6. August 27th, the obs. mer. alt. of *Vega* was 20° 45′ (Z.S.), index error +1′ 25″, height of the eye 22 feet; required the latitude.

Ans. Latitude 30° 41′ S.

LATITUDE BY MERIDIAN ALTITUDE OF A STAR UNDER THE POLE.

1. January 2nd, the obs. mer. alt. of *Achernar* under the *South* Pole was 20° 10′, I.E. −3′, height of the eye 14 feet; required the latitude.

Ans. Latitude 52° 5′ S.

2. June 10th, the obs. mer. alt. of *Canopus* under the S. Pole was 22° 14′, I.E. +1′ 40″, height of the eye 18 feet; required the latitude.

Ans. Latitude 59° 31′ S.

3. July 20th, the obs. mer. alt. of *η Argus* under the S. Pole was 30° 12′ I.E. −2′ 50″, height of the eye 20 feet; required the latitude.

Ans. Latitude 61° 4′ S.

4. August 14th, the obs. mer. alt. of α^1 *Crucis* under the S. Pole was 32° 22′, I.E. +1′, height of the eye 17 feet; required the latitude.

Ans. Latitude 59° 56′ S.

5. October 18th, the obs. mer. alt. of *α Ursæ Majoris* under the N. Pole was 18° 50′, I.E. −1′ 45″, height of the eye 22 feet; required the latitude.

Ans. Latitude 46° 13′ N.

6. November 14th, the obs. mer. alt. of *α Persei* under the N. Pole was 23° 55′, I.E. +2′ 30″, height of the eye 16 feet; required the latitude.

Ans. Latitude 64° 28′ N.

LATITUDE BY MERIDIAN ALTITUDE OF THE SUN.

1. September 20th, in long. 42° W., the obs. mer. alt. of the sun's L.L. was 50° 13′ 10″ (Z.N.), I.E. −2′ 10″, height of the eye 26 feet: required the latitude.

Declin. 1° 2′ 28″ N. Latitude 40° 41′ 14″ N.

2. September 21st, in long. 70° 30′ E., the obs. mer. alt. of the sun's L.L. was 60° 15′ 20″ (Z.S.), I.E. 0, height of the eye 23 feet; required the latitude.

Declin. 0° 46′ 25″ N. Latitude 28° 47′ 29″ S.

3. September 22nd, in long. 43° 30′ E., the obs. mer. alt. of the sun's L.L., by *artificial horizon*,* was 100° 14′ 20″ (Z.N.), I.E. +2′ 40″; required the latitude.

Declin. 0° 21′ 12″ N. Latitude 39° 57′ 25″ N.

4. September 23rd, in long. 70° 14′ W., the obs. mer. alt. of the sun's L. L., by *artificial horizon*, was 110° 25′ 10″ (Z.S.), I.E. −1′ 20″; required the latitude.

Declin. 0° 9′ 29″ S. Latitude 34° 42′ 11″ S.

5. September 23rd, in long. 80° 20′ E., the obs. mer. alt. of the sun's U.L., by *artificial horizon*, was 98° 30′ 40″ (Z.N.), I.E. + 1′ 10″; required the latitude.

Declin. 0° 0′ 21″ N. Latitude 41° 1′ 10″ N.

6. September 24th, in long. 4° 30′ W., the obs. mer. alt. of the sun's L.L., by *artificial horizon*, was 84° 14′ 30″ (Z.N.), I.E. −2′ 35″; required the latitude.

Declin. 0° 28′ 36″ S. Latitude 47° 10′ 26″ N.

LATITUDE BY AN ALTITUDE OF THE POLE STAR.

1. Sept. 21st, at 8h. 40m. P.M. mean time, in long. 33° W., the obs. alt. of the *Pole Star* was 50° 50′, index error +3′, height of the eye 23 feet; required the latitude.

R.A. merid. 20h. 42m.
1st corr. — 0° 32′ 51″ Latitude 50° 18′ N.

2. Sept. 23rd, at 2h. 45m. A.M. mean time, in long. 154° 30′ W., the obs. alt. *Polaris* was 49° 32′, index error — 2′ 55″, height of the eye 15 feet; required the latitude.

R.A. merid. 2h. 53m. 36s.
1st corr. — 1° 15′ Latitude 48° 9′ N.

3. September 12th, at 8h. 48m. P.M. mean time, in long. 45° W., the obs. alt. of the *Pole Star* was 45° 25′, I.E. −2′ 20″, height of the eye 17 feet; required the latitude.

R. A. mer. 20h. 15m.
1st corr. −23′ 30″. Latitude 44° 55′ N.

4. September 13th, at 4h. om. A.M. mean time, in long. 148° 30′ E., the obs. alt. of the *Pole Star* was 50° 15′, I.E. +1′ 30″, height of the eye 19 feet; required the latitude.

R.A. mer. 3h. 26m.
1st corr. −1° 10′. Latitude 49° 2′ N.

* See Note, page 77.

5. September 15th, at 9h. 40m. P.M. mean time, in long. 35° 30′ W., the obs. alt. of the *Pole Star* was 49° 40′, I.E. + 2′ 20″, height of the eye 24 feet; required the latitude.

R.A. mer. 21h. 19m.
1st corr. − 45′. Latitude 48° 53′ N.

6. September 17th, at 3h. 35m. A.M. mean time, in long. 48° W., the obs. alt. of *Polaris* was 35° 45′, I.E. − 3′ 30″, height of the eye 20 feet; required the latitude.

R.A. mer. 3h. 19m.
1st corr. — 1° 11′. Latitude 34° 24′ N.

VARIATION BY AMPLITUDE (SUN).

For Table of Deviations, see NAVIGATION, *p.* 47.

1. September 19th, at about 5h. 57m. P.M. (app. time), in lat. 22° 50′ S. and long. 38° 15′ W., the sun's setting amplitude by standard compass was W. 8° 15′ S., deviation 8° 50′ E.; required the variation.

Declin. 1° 20′ 9″ N. Corr. mag. amp. W. 0° 35′ N.
True amp. W. 1° 27′ N. Variation 0° 52′ E.

2. September 23rd, at 6h. om. A.M. (app. time), in lat. 39° 33′ N. and long. 160° 0′ W., the sun rose by standard compass E. 22° 0′ N., deviation 6° 0′ E.; required the variation.

Declin. 0° 9′ 24″ S. Corr. mag. amp. E. 16° 0′ N.
True amp. E. 0° 12′ S. Variation 16° 12′ E.

3. September 23rd, at about 6h. om. A.M. (mean time), in lat. 29° 11′ S. and long. 15° 30′ E., the sun rose by standard compass S. 68° 10′ E., ship's head being S.S.W.¼W. by standard compass; required the variation.

Declin. 0° 1′ 50″ N. Corr. mag. amp. E. 17° 5′ S.
True amp. E. 0° 2′ N. Variation 17° 7′ W.

4. September 20th, at 6h. A.M. mean time, in lat. 12° S. and long. 130° 30′ W., the sun's rising amplitude by standard compass was E. 10° S., ship's head west; find the variation.

Declin. 1° 2′ 28″ N. C.M.A. E. 4° N.
True amp. E. 1° 14′ N. Var. 2° 56′ E.

5. September 22nd, at 6h. A.M. mean time, in lat. 30° 15′ S. and long. 62° E., the sun rose by standard compass N. 68° E., ship's head N.E. by N.; find the variation.

Declin. 0° 28′ N. C.M.A. E. 17° N.
True amp. E. 0° 33′ N. Var. 16° 27′ E.

6. September 23rd, at 6h. P.M. mean time, in lat. 18° S. and long. 165° E., the sun set by standard compass N. 87° W., ship's head W. by S.; find the variation.

Declin. 0° 0′ 10″ S. C.M.A. W. 9° S.
True amp. W. 0° 0′ 30″ S. Var. 9° E.

7. September 23rd, at 6h. A.M. mean time, in lat. 50° 20′ N. and long. 3° 10′ W., the sun rose by standard compass S. 55° 32′ E., the variation at the place of the ship being 24° 30′ W.; find the deviation due to the direction of the ship's head,* which was W.S.W. by standard compass when the observation was taken.

Declin. 0° 0′ 37″ N. Deviation for
True amp. E. 0′ 45″ N. Ship's head, W.S.W. } 9° 59′ W.

VARIATION BY AZIMUTH.

1. September 24th, at 3h. 10m. P.M. mean time, in lat. 10° 15′ N. and long. 168° 0′ E., the obs. alt. sun's L.L. was 39° 28′ 0″, I.E. + 1′ 15″, height of the eye 18 feet, when it bore by standard compass S. 84° 20′ W., dev. 13° W.; required the variation.

P.D. 90° 20′ 38″. C.M.B. N. 108° 40′ 0″ W.
T.B. N. 99° 5′ 0″ W. Var. 9° 35′ 0″ E.

2. September 29th, at 2h. 40m. P.M. mean time, in lat. 49° 5′ N. and long. 40° W., the mean of a set of alts. of the sun's L.L. was 23° 8′ 40″ (index error + 4′ 50″), when the mean of the corresponding bearings of the sun's centre by azimuth compass was N. 78° 15′ W., height of the eye 18 feet, ship's head being W.S.W. by standard compass, and the correct variation 36° 30′ W.; find the deviation* due to the direction of the ship's head.

Pol. dist. 94° 22′ 42″. True bearing N. 128° 45′ W.
Deviation 10° 0′ W.

3. September 15th, at 4h. 15m. P.M. mean time, in lat. 19° 50′ N. and long. 38° 20′ W., the obs. alt. of the sun's L.L. was 25° 27′ 50″, I.E. + 2′, height of the eye 19 feet, when the sun bore by standard compass N. 93° 10′ W., dev. 6° 40′ E.; find the variation.

T.B. N. 96° 30′ W. Variation 10° W.
C.M.B. N. 86° 30′ W.

4. September 15th, at 3h. 15m. P.M. mean time in lat. 35° N. and long. 25° 30′ W., the obs. alt. of the sun's L.L. was 32° 39′ 50″, I.E. −1′ 25″, height of the eye 24 feet, when the sun bore by azimuth compass N. 77° 25′ W., dev. 12° W.; find the variation.

T.B. N. 112° 8′ W. Variation 22° 43′ W.
C.M.B. N. 89° 25′ W.

* See Note, page 73.

5. September 16th, at 4h. 25m. P.M. mean time, in lat. 24° 40′ S. and long. 50° 25′ W., the obs. alt. of the sun's L.L. was 17° 51′ 20″, I.E. + 2′ 30″, height of the eye 23 feet, when the sun bore by standard compass S. 81° 10′ W., ship's head by standard compass N.E. by E.½E.; find the variation.

T.B. S. 101° 33′ W. Variation 8° 23′ E.
C.M.B. S. 93° 10′ W.

6. Sept. 17th, at 4h. om. P.M. mean time, in lat. 15° N. and long. 21° W., the obs. alt. of the sun's L.L. was 27° 56′ 20″, I.E. + 2′ 30″, height of the eye 20 feet, when the sun bore by standard compass N. 93° 31′ W., the variation at the place of observation being 16° 10′ W.; find the deviation for the direction of the ship's head,* which was E. by N. at the time of taking the bearing.

T.B. N. 95° 41′ W. Deviation for E. by N. 14° E.

7. Sept. 20th, at 5h. 30m. P.M. mean time, in lat. 48° 45′ N. and long. 170° 20′ E., the mean of a set of alts. of the sun's L.L. was 15° 22′ 30″, index error — 2′ 10″, when the mean of a corresponding set of bearings of the sun's centre by azimuth compass was S. 49° 40′ W., height of the eye 20 feet, the ship's head being N. E. by standard compass, and the correct variation 10° 0′ E.; find the deviation of the compass for the direction of the ship's head.

Pol. dist. 88° 49′ 11″ True bearing N. 106° 27′ W.
Deviation 15° 53′ E.

LONGITUDE BY CHRONOMETER. (SUN.)

1. Sept. 15th, at about 4h. 15m. P.M. mean time, in lat. 19° 50′ N. and long. D.R. 39° 20′ W., the obs. alt. of the sun's L.L. was 25° 27′ 50″, I.E. + 2′, height of the eye 19 feet, when a chron. showed 6h. 39m. 45s.; find the longitude.

On August 23rd, at noon, the chron. was 8m. 15s. slow on G.M.T., and gaining daily 3·2s.

Declin. 2° 54′ 46″ N. M.T. ship 4h. 10m. 1s. Sept. 15th.
Eq. T. 4m. 56s. sub. M.T. chr. 6h. 46m. 46s. Sept. 15th.
Long. 39° 11′ 15″ W.

2. Sept. 15th, at about 3h. 15m. P.M. mean time, in lat. 35° N. and long. D.R. 24° 30′ W., the obs. alt. of the sun's L.L. was 32° 39′ 50″, I.E. — 1′ 25″, height of the eye 24 feet, when a chron. showed 0h. 59m. 10s.; find the longitude.

* See Note, page 73.

On Sept. 3d, at noon, the chron. was slow 3h. 57m. 50s. on G.M.T., and losing daily 4·4s.

Declin. 2° 56′ 32″ N. M.T. ship 3h. 20m. os. Sept. 15th.
Eq. T. 4m. 54s. sub. M.T. chr. 4h. 57m. 53s. Sept. 15th.
Long. 24° 28′ 15″ W.

3. Sept. 16th, at about 4h. 25m. P.M. mean time, in lat. 24° 40′ S. and long. D.R. 49° 25′ W., the obs. alt. of the sun's L.L. was 17° 51′ 20″, I.E. + 2′ 30″, height of the eye 23 feet, when a chron. showed 6h. 30m. 10s.; find the longitude.

Aug. 30th, at noon, the chron. was slow on G.M.T., 1h. 16m. 50s., and gaining daily 1·43s.

Declin. 2° 30′ 40″ N. M.T. ship 4h. 30m. 1s. Sept. 16th.
Eq. T. 5m. 18s. sub. M.T. chr. 7h. 46m. 35s. Sept. 16th.
Long. 49° 8′ 30″ W.

4. Sept. 17th, at about 4h. 0m. P.M. mean time, in lat. 15° N. and long. D.R. 21° W., the obs. alt. of the sun's L.L. was 27° 56′ 20″, I.E. + 2′ 30″, height of the eye 20 feet, when a chron. showed 5h. 10m. 15s.; find the longitude.

On Aug. 28th, at noon, the chron. was 14m. 4s. slow on G.M.T., and losing daily 1·7s.

Declin. 2° 9′ 43″ N. M.T. ship 4h. 0m. os. Sept. 17th.
Eq. T. 5m. 37s. sub. M.T. chr. 5h. 24m. 53s. Sept 17th.
Long. 21° 13′ 15″ W.

CHRONOMETER. (STAR, &c.)

1. Sept. 20th, at about 3h. 0m. A.M. mean time, in lat. 10° 12′ N. and long. D.R. 27° 30′ W., the obs. alt. of *Sirius* was 28° 38′ (E. of mer.), index error − 1′ 10″, height of eye 22 feet, when a chron. showed 0h. 44m. 30s.; required the longitude.

On Sept. 1st, at noon, the chron. was 4h. 5m. 12s. slow on G.M.T., and gaining 1·2s. daily.

R.A.M. sun 11h. 55m. 24s. M.T. chr. 16h. 49m. 19s. Sept. 19th.
Star's hour ang. 20h. 16m. 1s. Longitude 27° 18′ 45″ W.
M.T. ship 15h. 0m. 4s. Sept. 19th.

2. Sept. 15th, at about 8h. 25m. P.M. mean time, in lat. 35° 20′ S. and long. D.R. 50° 30′ E., the obs. alt. of the *Moon's* L.L. (W. of mer.) was 37° 17′ 30″, I.E. − 40″, height of the eye 26 feet, when a chron. showed 4h. 19m. 54s.; find the longitude.

On Aug. 29th, at noon, the chron. was 42m. 12s. slow on G.M.T., and losing 7·5s. daily.

Moon's hour angle 3h. 39m. 4s. M.T. chr. 5h. 4m. 14s. Sept. 15th.
M.T. ship 8h. 25m. 3s. Sept. 15th. Long. 50° 12′ 15″ E.

3. Sept. 16th, at about 8h. 10m. P.M. mean time, in lat. 44° 50′ N. and long. D.R. 54° 28′ W., the obs. alt. of *α Pegasi* was 40° 39′ (E. of mer.), I.E. +1′10″, height of the eye, 23 feet, when a chron. showed 0h. 48m. 17s.; find the longitude.

On Sept. 2nd, at noon, the chron. was 1h. 0m. 10s. fast on G.M.T., and losing daily 2·3s.

Star's hour angle 20h. 54m. 37s. M.T. chr. 11h. 48m. 39s. Sept. 16th.
M.T. ship 8h. 10m. 0s. Sept. 16th. Long. 54° 29′ 45″ W.

LATITUDE BY DOUBLE ALTITUDE.

1. Sept. 24th, in lat. D.R. 10° 25′ N. and long. 168° 15′ E., the following observations were taken to find the latitude:—

M. T. Ship.	Chron. times.	Obs. Alts. Sun's L.L.	True bearings.
11h. 15m. A.M.	0h. 5m. 15s.	75° 59′ 20″	S. 39° E.
3 10 P.M.	3 56 30	39 27 30	S. 82 W.

Index error +1′ 30″, height of the eye 18 feet. Run of the ship in the interval S. 30° E. 29 miles.

At G.B.		At L.B.	
P.D.	90° 20′ 38″	P.D.	90° 16′ 44″
Z.D.	50 20 16	Z.D.	13 18 52
Arc (1)	57 48 45		
(2)	90 6 45		
(3)	13 38 30	Polar angle 3h. 51m. 15s.	
(5)	65 46 45	Latitude at last obs. 10° 9′ N.	

2. September 17th, in lat. by D.R. 36° 20′ N., the following double altitude of two stars was observed at the same instant:—

Obs. Alt. *α Leonis.*	Bearing.	Obs. Alt. *α Orionis.*	Bearing.
45° 20′ 0″	S.E.	55° 11′ 0″	S.S.W.
Ind. error + 1 10		− 2 20	

Height of the eye 20 feet; required the latitude.

Arc (1) 62° 27′ 0″	Arc (3) 38° 5′ 15″	Pol. ang. 4h. 13m. 18s.
Arc (2) 88 11 15	Arc (4) 50 6 0	Latitude 36° 34′ N.

3. September 15th, in lat. D.R. 23° 20′ S. and long. 32° 19′ W., the following double altitude was observed :—

M.T. ship, nearly.	Times by chron.	Obs. alt. sun's L.L.	True bearings.
h. m.	h. m. s.	° ′ ″	
10 12 A.M.	0 23 3	53 27 40	N. 27° E.
2 25 P.M.	4 35 59	45 40 0	N. 58° W.

Height of the eye 24 feet, I.E. $-2'$ 25″, run of the ship in the interval W. by N. 6 miles ; find the latitude at the last observation.

Arc (1) 63° 8′ 15″ Arc (3) 39° 24′ 15″ Latitude 22° 47′ S.
Arc (2) 91 53 0 Arc (4) 52 28 45

4. Sept. 18th, in lat. D.R. 9° 40′ N. and long. 24° 16′ W., the following double altitude was taken :—

M.T. ship, nearly.	Times by chron.	Obs. alt. sun's L.L.	True bearings.
h. m.	h. m. s.	° ′ ″	
8 52 A.M.	10 54 26	43 53 30	S. 83° E.
11 20 A.M.	1 22 38	78 17 20	—

The run of the ship in the interval was N.E. by E. 15 miles, I.E. —40″, height of the eye 15 feet; find the latitude at the last observation.

Arc (1) 37° 1′ 45″ Arc (3) 11° 28′ 0″ Latitude 9° 55′ N.
Arc (2) 89 26 15 Arc (4) 77 58 15

5. September 19th, in lat. by account 26° 44′ N., the following altitudes of two stars were observed at the same moment :—

Obs. Alt. *β Orionis.*	Bearing.	Obs. Alt. *α Hydræ.*	Bearing.
39° 40′ 0″	S.W. by S.	47° 20′ 0″	S.E.
Ind. er. −2 10		+ 1 30	

Height of the eye 15 feet; required the latitude.

Arc (1) 62° 0′ 0″ Arc (3) 60° 50′ 45″ Pol. ang. 4h. 12m. 53s.
Arc (2) 95 12 15 Arc (4) 34 21 30 Latitude 26° 49′ N.

6. June 10th, in lat. D.R. 40° 22′ S., the following simultaneous altitudes of two stars were taken :—

Obs. alt. *Sirius.*	Bearing.	Obs. alt. *Spica.*	Bearing.
15° 25′	N. 99° W.	57° 1′	N. 57° W.

Index error + 1′ 35″, height of the eye 22 feet; required the latitude.

Arc (1) 96° 10′ 40″ Arc (3) 25° 20′ 15″ Latitude 40° 19′ S.
Arc (2) 77 7 45 Arc (4) 51 47 30

7. September 16th, in lat. D.R. 25° 30′ N., the following double altitude of two stars was taken :—

Obs. alt. *α Tauri.*	Bearing.	Obs. alt. *β Geminorum.*	Bearing.
74° 59′	S. 44° E.	37° 6′	S. 107° E.

Index error − 1′ 40″, height of the eye 30 feet; required the latitude.

Arc (1) 45° 2′ 0″ Arc (3) 17° 5′ 30″ Latitude 25° 31′ 52″ N.
Arc (2) 95 8 45 Arc (4) 78 3 15

8. September 17th, in lat. D.R. 54° 20′ N., the following simultaneous altitudes of two stars were taken :—

True alt. α *Andromedæ.*	Bearing.	True alt. α *Tauri.*	Bearing.
51° 18′	S. 24° E.	46° 46′	S. 33° E.
Arc (1) 62° 9′	Arc (3) 43° 22′	Latitude 54° 8′ N.	
Arc (2) 66 3	Arc (4) 22 41		

LATITUDE BY MERIDIAN ALTITUDE OF THE MOON.

1. September 15th, in long. 27° W., the obs. mer. alt. of the moon's L.L. was 54° 15′ 30″ (Z.N.), I.E. −3′ 30″, height of the eye 19 feet; required the latitude.

G.D. 6h. 36m. Sept. 15th.
Declin. 16° 52′ 50″ S. Latitude 18° 13′ 35″ N.

2. September 16th, in long. 35° W., the obs. mer. alt. of the moon's L.L. was 83° 10′ 20″ (Z.S.), I.E. + 2′ 10″, height of the eye 29 feet; required the latitude.

G.D. 7h. 57m. Sept. 16th.
Declin. 17° 58′ 51″ S. Latitude 24° 30′ S.

3. September 17th, in long. 32° E., the obs. mer. alt. of the moon's L.L. was 38° 17′ 40″ (Z.S.), I.E. − 50″, height of the eye 30 feet; required the latitude.

G.D. 4h. 8m. Sept. 17th.
Declin. 18° 10′ 25″ S. Latitude 69° 2′ 18″ S.

4. September 18th, in long. 45° W., the obs. mer. alt. of the moon's U.L. was 23° 18′ 20″ (Z.N.), I.E. + 1′ 40″, height of the eye 20 feet; required the latitude.

G.D. 10h. 15m. Sept. 18th.
Declin. 17° 24′ 47″ S. Latitude 48° 46′ 13″ N.

5. September 19th, in long. 54° E., the obs. mer. alt. of the moon's U.L. was 40° 10′ 30″ (Z.N.), I.E. −1′ 15″, height of the eye 20 feet; required the latitude.

G.D. 4h. 15m. Sept. 19th.
Declin. 16° 19′ 13″ S. Latitude 33° 9′ 38″ N.

LONGITUDE BY LUNAR.

1. September 20th, at 11h. 20m. P.M. mean time, in lat. 18° 50′ S. and long. by D.R. 90° E., the following lunar was taken :—

	Obs. alt. α *Arietis* (E. of mer.)	Obs. alt. moon's L.L.	Obs. dist. F.L.
	32° 36′ 30″	51° 0′ 0″	87° 24′ 25″
I.E.	+ 1 10	— 2 10	+ 0 20

height of the eye 28 feet; required the longitude.

Star's declin. 22° 49′ 50″ N. True dist. 86° 42′ 47″
Star's R.A. 1h. 59m. 41s. M.T. Green. 5h. 19m. 15s. Sept. 20th.
R.A.M. sun 11h. 57m. 28s. M.T. ship 11h. 20m. os. Sept. 20th.
Longitude 90° 11′ 15″ E.

2. September 23rd, at oh. 40m. A.M. mean time, in lat. 50° 15′ N. and long. by account 45° 8′ W., the following lunar was taken :—

	Obs. alt. *Aldebaran* (E. of mer.)	Obs. alt. moon's L.L.	Obs. dist. F.L.
	34° 28′ 40″	27° 22′ 50″	87° 51′ 10″
I.E.	— 1 20	+ 0 30	+ 1 5

height of the eye 20 feet; required the longitude.

Star's R.A. 4h. 28m. 17s. True dist. 87° 6′ 5″
Star's dec. 16° 14′ 14″ N. M.T. Green. 15h. 40m. 15s. Sept. 22nd.
R.A.M. sun 12h. 7m. 3s. M.T. ship 12h. 39m. 59s. Sept. 22nd.
Longitude 45° 4′ 0″ W.

3. September 15th, at 3h. 20m. P.M. mean time, in lat. 35° N. and long. D.R. 25° 30′ W., the following lunar was taken :—

	Obs. alt. sun's L.L.	Obs. alt. moon's L.L.	Obs. dist.
	32° 39′ 50″	33° 25′ 20″	74° 11′ 10″
I.E.	— 1 25	+ 1 40	+ 1 5

height of the eye 24 feet; required the longitude.

Declin. 2° 56′ 30″ N. M.T. ship 3h. 20m. os. Sept. 15th.
Eq. T. 4m. 54s. sub. M.T. Green. 5h. 4m. 43s. Sept. 15th.
True dist. 74° 23′ 18″. Longitude 26° 10′ 15″ W.

4. September 17th, at 4h. om. P.M. mean time, in lat. 15° N. and long. D.R. 21° W., the following lunar was taken :—

	Obs. alt. sun's L.L.	Obs. alt. moon's L.L.	Obs. dist.
	27° 56′ 20″	41° 33′ 40″	96° 20′ 10″
I.E.	+ 2 30	— 1 15	+ 1 0

height of the eye 20 feet; required the longitude.

Declin. 2° 9′ 43″ N. M.T. ship 4h. om. 1s. Sept. 17th.
Eq. T. 5m. 37s. sub. M.T. Green. 5h. 24m. 56s. Sept. 17th.
True dist. 96° 24′ 2″ Longitude 21° 13′ 45″ W.

5. September 16th, at 8h. 10m. P.M. mean time, in lat. 44° 50′ N. and long. D.R. 54° 30′ W., the following lunar was taken :—

Obs. alt. α *Pegasi* (E. of mer.)	Obs. alt. moon's L.L.	Obs. dist. F.L.
40° 38′ 50″	18° 0′ 10″	88° 35′ 30″
I.E. + 1 10	− 0 55	− 1 10

height of the eye 23 feet; required the longitude.

M.T. ship 8h. 9m. 58s. Sept. 16th. True dist. 87° 7′ 47″
M.T. Green. 11h. 47m. 18s. Sept. 16th. Long. 54° 20′ 0″ W.

6. September 17th, at 8h. 20m. P.M. mean time, in lat. 33° 55′ S. and long. D.R. 62° 30′ E., the following lunar was taken :—

Obs. alt. α *Pegasi* (E. of mer.)	Obs. alt. moon's L.L.	Obs. dist. F.L.
26° 35′ 10″	57° 36′ 0″	80° 32′ 30″
I.E. + 1 15	− 1 40	− 0 30

height of the eye 18 feet; required the longitude.

M.T. ship 8h. 20m. 0s. Sept. 17th. True dist. 80° 2′ 1″
M.T. Green. 4h. 10m. 4s. Sept. 17th. Long. 62° 29′ E.

FIXED STARS.

MEAN PLACES FOR JANUARY, 1866.

Star's Name.	Mag.	Right Ascension.	Annual Var.	Declination.	Annual Var.
		h. m. s.	s.	° ′ ″	s.
α Andromedæ	2	0 1 28	+3·1	N. 28 21 2	+19·9
γ Pegasi (*Algenib*)	3·2	0 6 20	3·1	N. 14 26 18	20·0
β Ceti	2	0 36 52	3·0	S. 18 43 22	19·8
α Ursæ Min. (*Polaris*)	2	1 9 58	19·6	N. 88 35 42	19·1
α Eridani (*Achernar*)	1	1 32 43	2·2	S. 57 55 5	18·4
α Arietis	2	1 59 37	3·4	N. 22 49 38	17·2
α Ceti	2·3	2 55 17	3·1	N. 3 33 43	14·4
α Persei	2	3 14 46	4·2	N. 49 22 52	13·2
α Tauri (*Aldebaran*)	1	4 28 14	3·4	N. 16 14 14	7·6
α Aurigæ (*Capella*)	1	5 6 48	4·4	N. 45 51 28	4·2
β Orionis (*Rigel*)	1	5 8 6	2·9	S. 8 21 33	4·5
β Tauri	2	5 17 49	3·8	N. 28 29 27	3·5
α Argus (*Canopus*)	1	6 20 59	1·3	S. 52 37 25	− 1·8
α Canis Maj. (*Sirius*)	1	6 39 15	2·6	S. 16 32 6	4·6
α^2 Gemin. (*Castor*)	2·1	7 26 3	3·8	N. 32 10 45	7·4
α Canis. Min. (*Procyon*)	1	7 32 17	3·1	N. 5 33 58	8·9
β Gemin. (*Pollux*)	1·2	7 37 7	3·7	N. 28 20 49	8·3
α Hydræ	2	9 21 0	2·9	S. 8 4 46	15·4
α Leonis (*Regulus*)	1·2	10 1 14	3·2	N. 12 37 15	17·4
η Argûs	2	10 39 52	2·3	S. 58 58 48	18·7
α Ursæ Majoris	2	10 55 26	3·8	N. 62 28 25	19·3
β Leonis	2	11 42 13	3·1	N. 15 19 16	20·1
α^1 Crucis	1	12 19 10	3·3	S. 62 21 18	19·9
α Virginis (*Spica*)	1	13 18 8	3·1	S. 10 27 40	18·9
η Ursæ Majoris	2	13 42 15	2·4	N. 49 58 59	18·1
β Centauri	1	13 54 24	4·1	S. 59 43 29	17·7
α Boötis (*Arcturus*)	1	14 9 33	2·7	N. 19 52 53	18·9
α^2 Centauri	1	14 30 32	4·0	S. 60 16 39	15·0
β Ursæ Minoris	2	14 51 0	−0·2	N. 74 42 10	14·7
α Coronæ Borealis	2	15 29 1	+2·5	N. 27 10 3	12·3
α Serpentis	2·3	15 37 40	2·9	N. 6 50 58	11·6
α Scorpii (*Antares*)	1·2	16 21 12	3·7	S. 26 7 54	8·4
α Trianguli Australis	2	16 34 30	6·3	S. 68 46 34	7·4
α Ophiuchi	2	17 28 43	2·8	N. 12 39 36	− 2·9
α Lyræ (*Vega*)	1	18 32 24	2·0	N. 38 39 39	+ 3·1
α Aquilæ (*Altair*)	1·2	19 44 15	2·9	N. 8 31 0	9·1
α Pavonis	2	20 15 2	4·8	S. 57 9 38	11·1
α Cygni	2·1	20 36 52	2·0	N. 44 48 10	12·7
α Gruis	2	21 59 46	3·8	S. 47 36 29	17·2
α Pis. Aus. (*Fomalhaut*)	1·2	22 50 14	3·3	S. 30 19 54	19·0
α Pegasi (*Markab*)	2	22 58 5	+3·0	N. 14 29 6	+19·3

AT APPARENT NOON, SEPTEMBER, 1866.

Day of the Month.	THE SUN'S				Equation of Time, *to be subtracted from Apparent Time.*	
	Apparent Right Ascension.	Diff. for 1 hour.	*Apparent* Declination.	Diff. for 1 hour.		Diff. for 1 hour.
	h. m. s.	s.	° ′ ″	″	m. s.	s.
1	10 41 33·64	9·072	N. 8 17 3·8	54·46	0 7·37	0·782
2	10 45 11·24	9·062	7 55 12·6	54·80	0 26·26	0·793
3	10 48 48·60	9·051	7 33 13·6	55·11	0 45·42	0·803
4	10 52 25·71	9·041	7 11 7·2	55·41	1 4·80	0·813
5	10 56 2·59	9·032	6 48 53·7	55·70	1 24·43	0·822
6	10 59 39·25	9·023	6 26 33·4	55·98	1 44·26	0·830
7	11 3 15·71	9·015	6 4 6·7	56·24	2 4·29	0·838
8	11 6 51·98	9·008	5 41 34·0	56·49	2 24·52	0·846
9	11 10 28·08	9·001	5 18 55·5	56·72	2 44·92	0·853
10	11 14 4·03	8·995	4 56 11·6	56·93	3 5·47	0·859
11	11 17 39·83	8·989	4 33 22·7	57·14	3 26·16	0·864
12	11 21 15·52	8·985	4 10 29·0	57·33	3 46·96	0·869
13	11 24 51·11	8·981	3 47 31·0	57·50	4 7·87	0·873
14	11 28 26·61	8·978	3 24 29·0	57·66	4 28·87	0·876
15	11 32 2·04	8·975	3 1 23·3	57·81	4 49·93	0·879
16	11 35 37·41	8·973	2 38 14·2	57·94	5 11·04	0·881
17	11 39 12·76	8·972	2 15 2·1	58·06	5 32·20	0·882
18	11 42 48·09	8·972	1 51 47·3	58·16	5 53·36	0·882
19	11 46 23·43	8·973	1 28 30·3	58·25	6 14·52	0·881
20	11 49 58·79	8·975	1 5 11·2	58·33	6 35·64	0·879
21	11 53 34·21	8·977	0 41 50·5	58·39	6 56·72	0·877
22	11 57 9·70	8·980	N. 0 18 28·4	58·44	7 17·73	0·873
23	12 0 45·27	8·985	S. 0 4 54·6	58·48	7 38·65	0·869
24	12 4 20·97	8·990	0 28 18·4	58·50	7 59·44	0·864
25	12 7 56·81	8·997	0 51 42·6	58·51	8 20·10	0·857
26	12 11 32·83	9·005	1 15 6·7	58·50	8 40·58	0·849
27	12 15 9·04	9·014	1 38 30·7	58·49	9 0·85	0·840
28	12 18 45·49	9·024	2 1 54·1	58·46	9 20·90	0·830
29	12 22 22·18	9·034	2 25 16·6	58·41	9 40·72	0·820
30	12 25 59·14	9·046	2 48 37·8	58·35	10 0·25	0·808
31	12 29 36·39	9·059	S. 3 11 57·4	58·28	10 19·50	0·796

AT MEAN NOON, SEPTEMBER, 1866.

Day of the Month.	THE SUN'S *Apparent* Right Ascension.	THE SUN'S *Apparent* Declination.	THE SUN'S Semidiam.*	Equation of Time, *to be added to Mean Time.*	Sidereal Time.
	h. m. s.	° ′ ″	′ ″	m. s.	h. m. s.
1	10 41 33·66	N. 8 17 3·7	15 53·6	0 7·37	10 41 41·03
2	10 45 11·31	7 55 12·2	15 53·8	0 26·27	10 45 37·59
3	10 48 48·71	7 33 12·9	15 54·0	0 45·43	10 49 34·14
4	10 52 25·87	7 11 6·2	15 54·3	1 4·82	10 53 30·69
5	10 56 2·80	6 48 52·4	15 54·5	1 24·45	10 57 27·24
6	10 59 39·51	6 26 31·8	15 54·7	1 44·28	11 1 23·79
7	11 3 16·02	6 4 4·8	15 54·9	2 4·32	11 5 20·34
8	11 6 52·34	5 41 31·7	15 55·2	2 24·55	11 9 16·90
9	11 10 28·49	5 18 52·9	15 55·4	2 44·96	11 13 13·45
10	11 14 4·49	4 56 8·7	15 55·7	3 5·51	11 17 10·00
11	11 17 40·35	4 33 19·4	15 55·9	3 26·21	11 21 6·56
12	11 21 16·09	4 10 25·4	15 56·2	3 47·02	11 25 3·11
13	11 24 51·73	3 47 27·1	15 56·4	4 7·93	11 28 59·66
14	11 28 27·28	3 24 24·7	15 56·7	4 28·93	11 32 56·21
15	11 32 2·76	3 1 18·6	15 57·0	4 50·00	11 36 52·76
16	11 35 38·19	2 38 9·2	15 57·3	5 11·12	11 40 49·31
17	11 39 13·59	2 14 56·7	15 57·5	5 32·28	11 44 45·87
18	11 42 48·97	1 51 41·6	15 57·8	5 53·45	11 48 42·42
19	11 46 24·36	1 28 24·2	15 58·1	6 14·61	11 52 38·97
20	11 49 59·78	1 5 4·8	15 58·4	6 35·74	11 56 35·52
21	11 53 35·25	0 41 43·7	15 58·6	6 56·82	12 0 32·07
22	11 57 10·79	N. 0 18 21·3	15 58·9	7 17·84	12 4 28·62
23	12 0 46·42	S. 0 5 2·1	15 59·2	7 38·76	12 8 25·18
24	12 4 22·17	0 28 26·2	15 59·5	7 59·56	12 12 21·73
25	12 7 58·06	0 51 50·7	15 59·7	8 20·22	12 16 18·28
26	12 11 34·13	1 15 15·2	16 0·0	8 40·70	12 20 14·83
27	12 15 10·40	1 38 39·5	16 0·3	9 0·98	12 24 11·38
28	12 18 46·90	2 2 3·2	16 0·6	9 21·03	12 28 7·93
29	12 22 23·64	2 25 26·0	16 0·8	9 40·85	12 32 4·49
30	12 26 0·65	2 48 47·5	16 1·1	10 0·39	12 36 1·04
31	12 29 37·95	S. 3 12 7·4	16 1·4	10 19·64	12 39 57·59

* The Semidiameter for *Apparent* Noon may be assumed the same as that for *Mean* Noon.

ELEMENTS from the NAUTICAL ALMANAC, 1866. MEAN TIME.

SEPTEMBER.

Day of the Month.	THE MOON'S Semidiameter.		Horizontal Parallax.		Meridian Passage.
	Noon.	*Midnight.*	*Noon.*	*Midnight.*	
	′ ″	′ ″	′ ″	′ ″	h. m.
15	14 47·3	14 48·0	54 11·0	54 13·5	4 45
16	14 49·5	14 51·6	54 18·7	54 26·4	5 32
17	14 54·4	14 57·9	54 36·7	54 49·5	6 20
18	15 2·0	15 6·8	55 4·8	55 22·3	7 9
19	15 12·2	15 18·0	55 41·9	56 3·3	7 58
20	15 24·2	15 30·8	56 26·2	56 50·2	8 48
21	15 37·5	15 44·3	57 14·9	57 39·8	9 39
22	15 51·0	15 57·5	58 4·4	58 28·2	10 29

ELEMENTS from the NAUTICAL ALMANAC, 1866. MEAN TIME.

LUNAR DISTANCES—SEPTEMBER.

Day of the Month.	Star's Name and Position.	*Noon.*	P.L. of diff.	III^h.	P.L. of diff.	VI^h.	P.L. of diff.	IX^h.	P.L. of diff.
		° ′ ″		° ′ ″		° ′ ″		° ′ ″	
15	Sun, W.....	72 5 33	3448	73 26 55	3448	74 48 17	3447	76 9 41	3445
16	Sun, W.....	82 57 23	3428	84 19 8	3423	85 40 58	3418	87 2 54	3412
16	*α* Pegasi, E.	93 19 52	3282	91 55 20	3277	90 30 42	3272	89 5 58	3267
17	Sun, W.....	93 54 29	3374	95 17 15	3365	96 40 11	3356	98 3 18	3345
17	*α* Pegasi, E.	82 0 43	3238	80 35 19	3231	79 9 47	3225	77 44 8	3219
20	*α* Arietis, E.	89 30 11	2811	87 55 57	2795	86 21 22	2779	84 46 26	2763
22	Antares, W.	84 23 12	2495	86 4 32	2479	87 46 14	2464	89 28 18	2449
22	Aldebaran, E.	96 6 36	2437	94 23 54	2422	92 40 51	2406	90 57 25	2391

ELEMENTS from the NAUTICAL ALMANAC, 1866. MEAN TIME.

THE MOON'S RIGHT ASCENSION AND DECLINATION.

Hour.	Right Ascension.	Declination.	Diff. Dec. for 10m.	Hour.	Right Ascension.	Declination.	Diff. Dec. for 10m.
	SEPTEMBER 15TH.				SEPTEMBER 18TH.		
	h. m. s.	° ′ ″	″		h. m. s.	° ′ ″	″
5	16 23 32·6	16 47 5·3 S.	36·44	10	19 4 59·6	17 25 29·0 S.	28·25
6	16 25 35·2	16 50 41·7	35·68	11	19 7 8·2	17 22 36·9	29·13
7	16 27 37·9	16 54 13·5	34·92		SEPTEMBER 19TH.		
	SEPTEMBER 16TH.			4	19 43 44·0	16 20 19·4 S.	44·12
7	17 17 9·1	17 55 32·6 S.	15·90	5	19 45 53·6	16 15 52·1	44·99
8	17 19 14·0	17 57 5·5	15·08		SEPTEMBER 23RD.		
	SEPTEMBER 17TH.			22	23 55 9·0	0 7 37·6 S.	111·84
4	18 1 9·7	18 10 27·2 S.	1·85	23	23 57 25·3	0 3 33·7 N.	111·93
5	18 3 16·3	18 10 13·5	2·72				

ELEMENTS from the NAUTICAL ALMANAC, 1866. MEAN TIME.

LUNAR DISTANCES—SEPTEMBER.

Day of the Month	Star's Name and Position.	*Midnight.*	P.L. of diff.	XV^h.	P.L. of diff.	XVIII^h.	P.L. of diff.	XXI^h.	P.L. of diff.
		° ′ ″		° ′ ″		° ′ ″		° ′ ″	
15	Sun, W.....	77 31 7	3442	78 52 36	3440	80 14 8	3437	81 35 43	3432
16	Sun, W.....	88 24 57	3405	89 47 8	3399	91 9 26	3391	92 31 53	3383
16	*α* Pegasi, E.	87 41 8	3262	86 16 12	3256	84 51 9	3251	83 26 0	3244
17	Sun, W.....	99 26 37	3336	100 50 7	3325	102 13 50	3313	103 37 47	3301
17	*α* Pegasi, E.	76 18 21	3213	74 52 27	3206	73 26 25	3199	72 0 14	3193
20	*α* Arietis, E.	83 11 9	2747	81 35 31	2731	79 59 32	2716	78 23 13	2700
22	Antares, W.	91 10 43	2434	92 53 30	2419	94 36 38	2405	96 20 6	2391
22	Aldebaran, E.	89 13 37	2376	87 29 27	2362	85 44 57	2347	84 0 5	2333

TABLES FOR DETERMINING THE LATITUDE BY OBSERVATIONS OF THE POLE STAR OUT OF THE MERIDIAN.

TABLE I. (A.)

Containing the *First* Correction.

Argument: Sidereal Time of Observation.

Sidereal Time of Obs.	Correction.	Sidereal Time of Obs.	Sidereal Time of Obs.	Correction.	Sidereal Time of Obs.
h. m.	° ′ ″	h. m.	h. m.	° ′ ″	h. m.
2 0	− 1 22 1 +	14 0	8 0	+ 0 18 11 −	20 0
10	1 21 8	10	10	0 21 44	10
20	1 20 7	20	20	0 25 16	20
30	1 18 56	30	30	0 28 44	30
40	1 17 36	40	40	0 32 9	40
50	1 16 8	50	50	0 35 30	50
3 0	1 14 31	15 0	9 0	0 38 47	21 0
10	1 12 45	10	10	0 42 0	10
20	1 10 51	20	20	0 45 8	20
30	1 8 49	30	30	0 48 11	30
40	1 6 39	40	40	0 51 8	40
50	1 4 21	50	50	0 54 0	50

TABLE II. (A.)

Containing the *Second* Correction (*always to be added*).

Arguments:—Sidereal Time and Altitude.

Sidereal Time of Obs.	ALTITUDE.								Sidereal Time of Obs.
	15°	20°	25°	30°	35°	40°	45°	50°	
h. m.	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	h. m.
2 0	0 1	0 1	0 1	0 2	0 2	0 2	0 3	0 3	14 0
3 0	0 4	0 5	0 6	0 8	0 9	0 11	0 13	0 16	15 0
4 0	0 8	0 10	0 13	0 16	0 20	0 24	0 28	0 33	16 0
5 0	0 12	0 16	0 20	0 25	0 31	0 37	0 44	0 52	17 0
6 0	0 15	0 20	0 26	0 32	0 39	0 47	0 56	1 7	18 0
7 0	0 16	0 22	0 29	0 36	0 43	0 52	1 1	1 13	19 0
8 0	0 16	0 21	0 27	0 34	0 41	0 49	0 59	1 10	20 0
9 0	0 13	0 18	0 23	0 28	0 34	0 41	0 48	0 58	21 0
10 0	0 9	0 12	0 16	0 19	0 23	0 28	0 33	0 40	22 0

TABLE III. (A.)

Containing the *Third* Correction (*always to be added*).

Arguments:—Sidereal Time and Date.

Sidereal Time of Obs.	March 1.	April 1.	June 1.	Sept. 1.
h.	′ ″	′ ″	′ ″	′ ″
2	0 58	0 49	0 32	0 36
4	1 1	0 57	0 39	0 28
14	1 2	1 11	1 28	1 24
18	0 56	0 55	1 9	1 32
20	0 54	0 48	0 54	1 23
22	0 53	0 44	0 41	1 7

LONGITUDE BY CHRONOMETER.

1. September 10th, at about 8h. 10m. A.M., in lat. 30° 10′ S. and long. D.R. 45° 30′ W., when a chron. showed 0h. 30m. 10s., the obs. alt. of the sun's L.L. was 24° 40′ 30″, index error +1′ 30″, height of the eye 20 feet: required the longitude.

August 20th, the chron. was fast on G.M.T. 1h. 20m. 10s., and on August 30th, it was fast on G.M.T. 1h. 20m. 30s.

See (*Note*).—The rate to be ascertained from these two errors.

Ans. Long. 45° 31′ 30″ W.

2. September 23rd, at about 8h. 10m. A.M., in lat. 39° 50′ N., and long. D.R. 152° E., when a chron. showed 5h. 30m. 25s., the obs. alt. of the Sun's L.L. was 21° 52′ 15″, index error +2′ 10″, height of the eye 24 feet: required the longitude.

August 15th, the chron. was slow on G.M.T. 4h. 12m. 30·5s., and on August 25th, it was slow on G.M.T. 4h. 12m. 5·5s.

Ans. Long. 151° 57′ E.

3. September 27th, at 3h. 30m. P.M., in lat. 49° 51′ N., and long. D.R. 96° 30′ W., when a chron. showed 1h. 30m. 20s., the obs. alt. of Sun's L.L. was 19° 50′ 30″, index error +2′ 30″, height of the eye 18 feet: required the longitude.

July 30th, at noon, the chron. was fast on G.M.T. 3h. 30m. 40s., and on August 29th, it was fast on G.M.T. 3h. 31m. 10·5s.

Ans. Long. 96° 38′ 30″ W.

4. September 17th, at about 7h. 0m. A.M., in lat. 20° 10′ N., and long. D.R. 37° 30′ E., the following observation for longitude was taken:—

Time by Chron.	Obs. Alt. Sun's L.L.
3h. 7m. 10s.	16° 10′ 10″
Index error +1′ 10″.	Dip, 20 feet.

On September 2nd, at noon, the chron. was slow on G.M.T. 1h. 22m. 19s., and on September 10th, it was slow on G.M.T. 1h. 22m. 59s.

Ans. Long. 37° 30′ E.

5. September 23rd, at about 7h. 0m. A.M., in lat. 34° 0′ N., and long. D.R. 1° 0′ W., the following observation was taken to determine the longitude:—

Time by Chron.	Obs. Alt. Sun's L.L.
5h. 56m. 55s.	15° 53′ 0″
Index error −1′ 10″	Dip, 18 feet.

On August 30th, at noon, the chron. was slow on G.M.T. 1h. 11m. 48s., and on September 15th, it was slow on G.M.T. 1h. 10m. 6s.

Ans. Long. 1° 1′ W.

6. September 10th, at about 7h. 30m. A.M., in lat. 32° S., and long. D.R. 68° 15′ W., the following observation was taken to determine the longitude:—

Time by Chron.	Obs. Alt. Sun's L.L.
11h. 57m. 48s.	15° 51′ 10″
Index error +0′ 30″.	Dip, 19 feet.

On August 10th, at noon, the chron. was fast on G.M.T. 0h. 0m. 54s., and on August 30th, it was slow on G.M.T. 0h. 1m. 6s.

Ans. Long. 68° 25′ 15″ W.

ERROR OF CHRONOMETER.

1. September 14th, A.M., in lat. 50° 21′ 36″ N., and long. 3° 35′ W. the following sights were taken to find the error of chronometer :—

Time by Chron.	Obs. Alt. Sun's L.L. (art. hor.)
8h. 47m. 0s.	53° 48′ 50″
8 47 20	53 53 40
8 47 39	53 58 40
8 47 54	54 3 30
8 48 10	54 7 30

Index error +0′ 45″.

Ans. Error, 3m. 7·6s. slow on G.M.T.

2. September 13th, A.M., in lat. 50° 21′ 36″ N., and long. 3° 35′ W., the following sights were taken to find the error of chronometer :—

Time by Chron.	Obs. Alt. Sun's L.L. (art. hor.)
9h. 21m. 14s.	63° 13′ 20″
9 21 25	63 16 10
9 21 35	63 18 10
9 21 48	63 21 20
9 21 59	63 24 0

Index error 0.

Ans. Error, 2m. 44·2s. slow on G.M.T.

3. September 25th, in lat. 53° 21′ 36″ N., and long. 3° 35″ W., the following observation was taken to find the error of the chronometer :—

Time by Watch.	Obs. Alt. Sun's L.L. (art. hor.)
9h 31m. 27s. (A.M.)	57° 39′ 40″

Index error − 0′ 30″.

Watch fast of chron. 1h. 42m. 18s. On September 13th, chron. was slow on G.M.T. 1h. 38m. 41s.

Find the error and also the rate.

Ans. Error, 1h. 41m. 33s. slow on G.M.T.

Rate, 14·33 seconds, losing.

Note.—When the errors on two different days are given, from which the daily rate has to be determined, if the errors are of the *same* name, that is, both *fast*, or both *slow*, it is merely necessary to take one error from the other, and divide the *difference* by the number of days elapsed between the dates of the two errors, the quotient will be the daily rate.

(*a*) If both errors are *fast*, the rate will be *gaining* when the *last* error was the greater ; and *losing* when the *last* error was the lesser of the two.

(*b*) If both errors are *slow*, the rate will be *losing* when the *last* error was the greater ; and *gaining* when the *last* error was the lesser of the two.

(*c*) When the errors on the two given days are of *contrary* names, that is, when one is fast and the other slow, or when one is slow and the other fast, the "rate" will be ascertained by dividing the *sum* of the errors by the number of days elapsed between the dates of the two errors.

(*d*) The "rate" will be *gaining* if the chronometer has changed from slow to fast ; and *losing* if the chronometer has changed from fast to slow.

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APPENDIX:

CONTAINING

EXPLANATIONS AND PROOFS

OF THE

RULES.

Principal Formulæ used in the following Investigations.

PLANE TRIGONOMETRY.

$$\text{Sin. } \overline{90^\circ - A} = \text{Cos. A} \quad \text{(i)} \quad : \quad \text{Cos. } \overline{90^\circ - A} = \text{Sin. A} \quad \text{(ii)}$$

$$\text{Tan. } \overline{90^\circ - A} = \text{Cot. A} \quad \text{(iii)} \quad : \quad \text{Cot. } \overline{90^\circ - A} = \text{Tan. A} \quad \text{(iv)}$$

$$\text{Sin. } \overline{90^\circ + A} = \text{Cos. A} \quad \text{(v)} \quad : \quad \text{Cos. } \overline{90^\circ + A} = -\text{Sin. A} \quad \text{(vi)}$$

$$\text{Sin. } \overline{180^\circ - A} = \text{Sin. A} \quad \text{(vii)} \quad : \quad \text{Cos. } \overline{180^\circ - A} = -\text{Cos. A} \quad \text{(viii)}$$

$$\frac{1}{\text{Sin. A}} = \text{Cosec. A} \quad \text{(ix)} \quad : \quad \frac{1}{\text{Cos. A}} = \text{Sec. A} \quad \text{(x)}$$

$$\frac{1}{\text{Cot. A}} = \text{Tan. A} \quad \text{(xi)}$$

$$\text{Sin. (A + B)} = \text{Sin. A . Cos. B} + \text{Cos. A . Sin. B} \quad (12)$$

$$\text{Sin. (A - B)} = \text{Sin. A . Cos. B} + \text{Cos. A . Sin. B} \quad (13)$$

$$\text{Cos. (A + B)} = \text{Cos. A . Cos. B} - \text{Sin. A . Sin. B} \quad (14)$$

$$\text{Cos. (A - B)} = \text{Cos. A . Cos. B} + \text{Sin. A . Sin. B} \quad (15)$$

$$\text{Sin. A} - \text{Sin. B} = 2\,\text{Cos.} \frac{\text{A + B}}{2} \text{. Sin.} \frac{\text{A - B}}{2} \quad (16)$$

$$\text{Cos. B} - \text{Cos. A} = 2\,\text{Sin.} \frac{\text{A + B}}{2} \text{. Sin.} \frac{\text{A - B}}{2} \quad (17)$$

$$\text{Cos. (A + B)} + \text{Cos. (A - B)} = 2\,\text{Cos. A . Cos. B} \quad (18)$$

$$1 - \text{Cos. A} = \text{Vers. A, or 2 Hav. A} \quad (19)$$

$$\text{Sin.} \frac{\text{A}}{2} = \sqrt{\text{Hav. A}} \quad (20)$$

SPHERICAL TRIGONOMETRY.

$$\text{Cos. A} = \frac{\text{Cos. } a - \text{Cos. } b \text{ . Cos. } c}{\text{Sin. } b \text{ . Sin. } c} \quad (a)$$

$$\text{Sin. } a \text{ : Sin. } b \text{ :: Sin. A : Sin. B} \quad (b)$$

Napier's Rules for the solution of right-angled triangles.

APPENDIX.

EXPLANATIONS AND PROOFS OF THE RULES, &c.,

IN

NAVIGATION AND NAUTICAL ASTRONOMY.

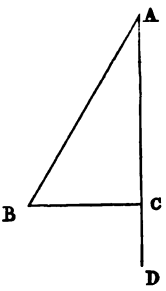
IN those problems in Navigation in which we are confined to distances that do not extend beyond a day's run, we may, for all practical purposes, consider the earth's surface to be a plane, and we may apply the rules of plane trigonometry to solve what are in reality spherical triangles, without any material errors resulting from this assumption.

PLANE SAILING.

I.—*To prove that dep.* = *dist.* × *Sin. course,* (1)

and diff. lat. = *dist.* × *Cos. course.* (2)

Fig. 1.



Let AD be a portion of a meridian, and suppose a ship to have sailed a distance AB, making an angle BAC with the meridian, this angle will be the course. Drop the perpendicular BC on AD; then as BC is perpendicular to the meridian AD, it represents a part of a parallel of latitude, and as it is also the distance of the ship from the meridian left, it must represent the departure, and AC, being a portion of the meridian included between the parallels of A and B, will represent the difference of latitude.

Now, BC = AB . Sin. BAC

and AC = AB . Cos. BAC;

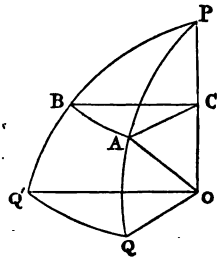
or, dep. = dist. × Sin. course. (1)

and diff. lat. = dist. × Cos. course. (2)

PARALLEL SAILING.

II.—*To prove that dep. = diff. long. × Cos. lat.* (3)

Fig. 2.



Suppose a ship to sail on a parallel of latitude from A to B, then will AB represent her departure, and QQ′, the corresponding arc of the equator, her difference of longitude. Let P be the Pole and O the centre of the earth, and join OP, OQ, OQ′; draw AC parallel to QO, and BC parallel to Q′O, and join AO. Then will angles POQ and OCA be right angles, and the angle CAO = alt. angle AOQ = the arc AQ, or = the latitude of AB.

Now, since the angles BCA, QOQ′ are equal, and arcs of circles subtending equal angles are proportional to their radii (Euc. B. VI., p. 33).

$$\therefore \frac{AB}{QQ'} = \frac{AC}{OQ} = \frac{AC}{AO} = \text{Cos. CAO} = \text{Cos. AOQ} = \text{Cos. lat.}$$

$$\therefore \frac{\text{dep.}}{\text{diff. long.}} = \text{Cos. lat.}$$

$$\text{or,} \quad \text{dep.} = \text{diff. long.} \times \text{Cos. lat.} \qquad (3)$$

Cor. Since $\frac{AB}{QQ'}$ or $\frac{\text{dep.}}{\text{diff. long.}} = \frac{AC}{AO}$, and Cos. CAO = Cos. lat., it is evident that in the plane triangle CAO, we may take AC to represent the departure, AO the difference of longitude, and the angle CAO the latitude, and we can solve all questions that may occur in this rule by means of this plane triangle.

MIDDLE LATITUDE SAILING.

III.—*To prove that dep. = diff. long. × Cos. mid. lat.* (4)

Fig. 3.

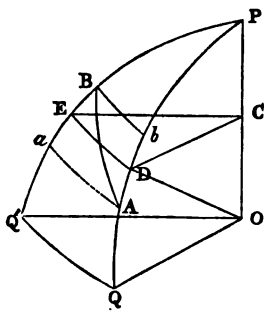
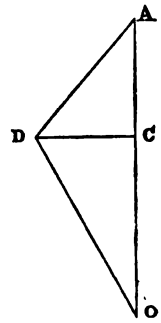


Fig. 4.



Suppose a ship to sail from A to B, but *not* on a parallel of latitude, then it is evident that if A*a* be taken for the departure it will be too great; and if B*b* be taken for it, it will be as much too small; therefore ED, which is an arc of a parallel at the middle latitude between A and B, will be the true departure, nearly, and we shall have, as in the last case,—

$$\frac{DE}{QQ'} = \frac{DC}{OQ} = \frac{DC}{OD} = \text{Cos. CDO} = \text{Cos. mid. lat.}$$

$$\text{or, } \frac{\text{dep.}}{\text{diff. long.}} = \text{Cos. mid. lat.}$$

$$\therefore \text{ dep.} = \text{diff. long.} \times \text{Cos. mid. lat.}$$

Cor. I.—It is evident that, as in the last rule, CD, DO, and angle CDO of the plane triangle CDO, will represent the departure, difference of longitude, and middle latitude respectively.

Cor. II. (FIG. 4).—Since the parts of the triangle CDO represent the above-mentioned elements, if OC be produced to A, and AC be made equal to the difference of latitude

between A and B, and AD, be joined, then will AD represent the distance, and DAC the course; and we shall have

$$\text{Tan. DAC} = \frac{\text{DC}}{\text{AC}} = \frac{\text{DO . Cos. ODC}}{\text{AC}}$$

$$\text{or, Tan. course} = \frac{\text{diff. long.} \times \text{Cos. mid. lat.}}{\text{diff. lat.}} \qquad (5)$$

$$\text{Also, AD} = \text{AC . Sec. DAC}$$

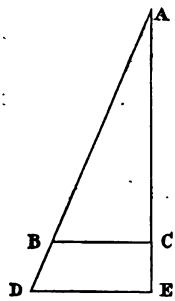
$$\text{or, dist.} = \text{diff. lat.} \times \text{Sec. course.} \qquad (6)$$

MERCATOR'S SAILING.

IV.—*To prove that Tan. course* $= \frac{\textit{diff. long.}}{\textit{Mer. diff. lat.}}$ (7)

and dist. $=$ *diff. lat.* $\times$ *Sec. course.* (8)

Fig. 5.



In the figure, let the parts of the triangle ABC represent the same elements as in plane sailing, and let DE be the difference of longitude corresponding to the departure BC; then if AE be taken, so that AE : AC :: DE : BC, AE will be the meridional difference of latitude between A and B.

$$\text{Now, Tan. BAC} = \frac{\text{DE}}{\text{AE}}$$

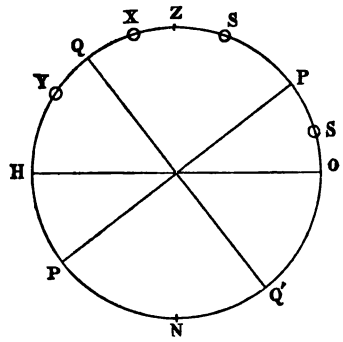
$$\text{or, Tan. course} = \frac{\text{diff. long.}}{\text{mer. diff. lat.}}$$

$$\text{Also, AB} = \text{AC . Sec. BAC} \qquad (7)$$

$$\text{or, dist.} = \text{true diff. lat. Sec. course.} \qquad (8)$$

V.—*To prove the Rule for finding the Latitude by the Meridian Altitude of a Heavenly Body.*

Fig. 6.



Let ZHNO be the celestial meridian, HO the horizon, QQ′ the equator, Z the Zenith, PP′ the poles.

(i) Let the declination be of the *same* name as the latitude, and suppose the body to be at X, south of the zenith,

Then $ZQ = ZX + QX = (90° - HX) + QX$,

or, lat. = (90 − alt.) + dec. = zenith dist. + decl.

(ii) Let the latitude and declination be of *different* names, and suppose the body to be at Y,

Then $ZQ = ZY - QY = (90° - HY) - QY$,

or, lat. = (90 − alt.) − dec. = zenith dist. − dec.

(iii) Let the body be north of the zenith, as at S,

Then $ZQ = QS - ZS = QS - (90 - SO)$,

or, lat. = dec. − (90 − alt.) = dec. − zenith dist.

VI.—*To prove the Rule for finding the Latitude by the Meridian Altitude of a Heavenly Body below the Pole.*

Let S′ in the above figure be the body below the pole,

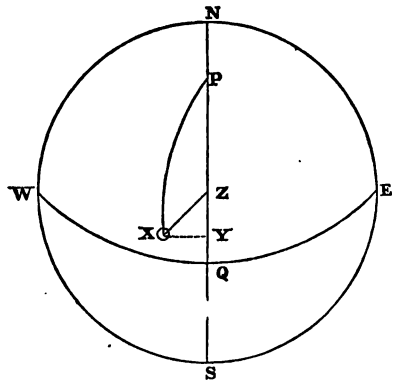
Then $PO = S'O + PS' = S'O + (90 - S'Q')$,

or, lat. = alt. + 90 − dec.

VII.—*To explain and prove the Rule for finding the Latitude by an Altitude of the Sun near the Meridian.*

FIRST METHOD.

Fig. 7.



Let NWSE represent the horizon, NZS the celestial meridian, &c.; let X be the place of the sun near the meridian, and let ZPX, or h be the sun's hour angle found by a chronometer whose error on ship mean time is known (nearly). Drop the perpendicular XY on the meridian, and let PX $= \overline{90-d}$, ZX $= \overline{90-a}$, PY $= x$, and ZY $= y$, XY $= z$. Then it is required to find from the above data, x and y, when we shall have $x - y =$ PZ, the Colatitude:

Now, in Δ PXY,

$$\text{Cos. } h = \text{Cot } \overline{90-d} \text{ . Tan. } x = \text{Tan. } d \text{ . Tan. } x$$

$$\therefore \text{Tan. } x = \text{Cos. } h \text{ . Cot. } d. \qquad (1)$$

From this formula we obtain x, and also Sec. x.

Again, $\text{Cos. } \overline{90-d} = \text{Cos. } x \text{ . Cos. } z.$

or, $$\text{Sin. } d = \text{Cos. } x \text{ . Cos. } z. \qquad (2)$$

Also in Δ ZXY, $\text{Cos. } \overline{90-a} = \text{Cos. } y \text{ . Cos. } z.$

or, $$\text{Sin. } a = \text{Cos. } y \text{ . Cos. } z. \qquad (3)$$

From (2) Cos. z = Sin. d . Sec. x, and from (3) Cos. z = Sin. a . Sec. y.

$$\therefore \text{Sin. } a \text{ . Sec. } y = \text{Sin. } d \text{ . Sec. } x.$$

$$\therefore \text{Sec. } y = \text{Cosec. } a \text{ . Sin. } d \text{ . Sec. } x,$$

which determines y.

Lastly, $x - y =$ PZ, the Colatitude.

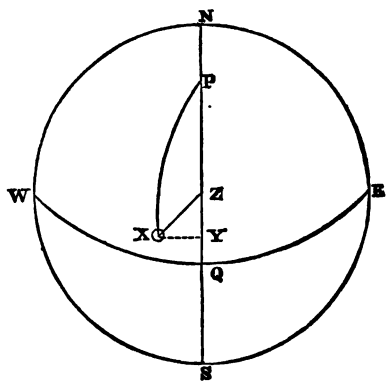
But if XY fall between Z and P, then $x + y =$ the Colat.

NOTE.—If the latitude and declination be of different names, PX will equal $90° + d$, in which case, as Cot. $\overline{90° + d}$ is negative, we must subtract the arc given by Formula (1) from 180° to obtain x.

VIII.—*To explain and prove the Rule for finding the Latitude by an Altitude of the Sun near the Meridian.*

SECOND METHOD.

Fig. 8.



Making the same construction as before, with P as pole, describe the arc XY; it is required to reduce the altitude of the sun at X to what it would have been at Y, *on the meridian*, and thus to obtain ZY, the meridian zenith distance, then by applying the declination, as in the meridian altitude, to obtain the latitude.

$$\text{In } \triangle \text{ PXZ, Cos. } h = \frac{\text{Cos. } z - \text{Cos. } \overline{90-d} \text{ . Cos. } \overline{90-l}}{\text{Sin. } \overline{90-d} \text{ . Sin. } \overline{90-l}} \quad (a)$$

$$= \frac{\text{Cos. } z - \text{Sin. } d \text{ . Sin. } l}{\text{Cos. } d \text{ . Cos. } l} \quad \text{(i and ii)}$$

Subtracting each side from 1,

$$1 - \text{Cos. } h = 1 - \frac{\text{Cos. } z - \text{Sin. } d \text{ . Sin. } l}{\text{Cos. } d \text{ . Cos. } l}$$

$$i.\,e.\ \text{Vers. } h = \frac{\text{Cos. } d \text{ . Cos. } l + \text{Sin. } d \text{ . Sin. } l - \text{Cos. } z}{\text{Cos. } d \text{ . Cos. } l.} \quad (19)$$

$$\text{Or, } 2 \text{ Hav. } h = \frac{\text{Cos. } (d-l) - \text{Cos. } z}{\text{Cos. } d \text{ . Cos. } l} \quad (15 \text{ and } 19)$$

$$\therefore 2 \text{ Cos. } d \text{ . Cos. } l \text{ . Hav. } h = \text{Cos. } (d-l) - \text{Cos. } z$$

$$- \text{Cos. } (d-l) = - \text{Cos. } z - 2 \text{ Cos. } d \text{ . Cos. } l \text{ . Hav. } h.$$

Adding 1 to both sides,

$$1 - \text{Cos. } (d-l) = 1 - \text{Cos. } z - 2 \text{ Cos. } d \text{ . Cos. } l \text{ . Hav. } h$$

$$\text{or, Vers. } (d-l) = \text{Vers. } z - 2 \text{ Cos. } d \text{ . Cos. } l \text{ . Hav. } h.$$

NOTE.—The Trigonometrical Formulæ referred to in this, and in the following investigations, are to be found in the list subjoined.

But $(d - l)$ or $(d + l)$ if lat. and dec. be of different names, is the meridian zenith distance ; Z, suppose,

$$\therefore \text{Vers. } Z = \text{Vers. } z - 2 \text{ Cos. } d \text{ . Cos. } l \text{ . Hav. } h.$$

$$\text{or, } \quad \text{Vers. } Z = \text{Vers. } z - \text{Vers. } \theta$$

$$\text{where Vers. } \theta = 2 \text{ Cos. } d \text{ . Cos. } l \text{ . Hav. } h$$

or, in logs.,

$$\log. \left(\frac{\text{Vers. } \theta}{1000000}\right)^{*} = \log. 2 + \log. \text{Cos. } d + \log. \text{Cos. } l + \log. \text{Hav. } h - 30$$

$$i.e. \quad \log. \text{Vers. } \theta - 6 = \cdot 301030 + \log. \text{Cos. } d + \log. \text{Cos. } l + \log. \text{Hav. } h - 30.$$

Whence

$$\log. \text{Vers. } \theta = 6 \cdot 301030 + \log. \text{Cos. } d + \log. l + \log. \text{Hav. } h - 30.$$

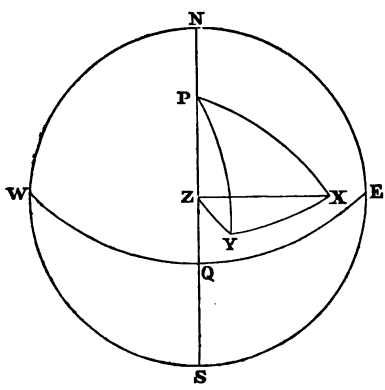
Which in words is the rule for finding Vers. θ ; this subtracted from Vers. z, gives Vers. Z, the meridian zenith distance:—

Lastly, Z $\sim$ d = lat.

* All the tabular Versines are, for convenienee sake, multiplied by one million ; therefore, when using them as above, we must be careful to divide them by 1000000.

IX.—*To explain the Rule for finding the Latitude by Sun Double Altitude.*

Fig. 9.



Let NWSE be the horizon, NZS the meridian, WQE the equator, and let X be the place of the sun at the greater bearing, Y the place of the sun at the less bearing. Join XY by the arc of a great circle, and draw PX, PY, the two polar distances, and ZX, ZY, the two zenith distances.

Then, in triangle PXY, we have given PX, PY, the two polar distances, and the included angle YPX, which is the elapsed time between the observations, to find XY. This is called Arc 1.

In the same triangle, with XY and the other two sides PX, PY, we can find PXY, or Arc 2.

Again, in the triangle ZXY, with XY and the two zenith distances ZX, ZY, we can find the angle ZXY. This is Arc. 3.

Then PXY − ZXY, or Arc 2 − Arc 3 = PXZ, or Arc 4.

Lastly, in the triangle PXZ, with the two sides, PX, ZX, and the included angle PXZ, we can find PZ, the colatitude.

Then 90° − PZ = latitude.

Fig. 10.

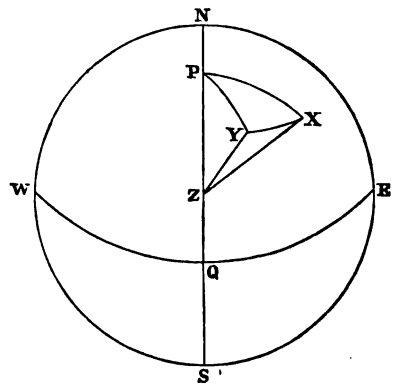
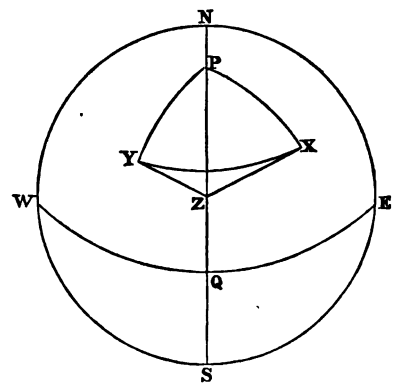


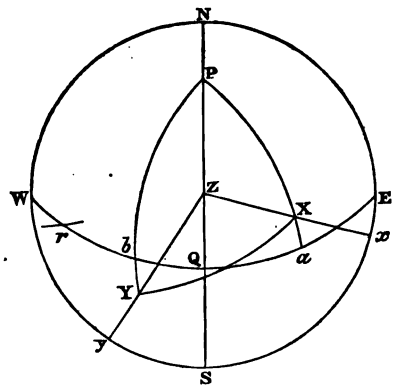
Fig. 11.



Cases may arise in which the arc joining X and Y will pass between the pole and zenith, as in Fig 11, or being produced will do so as in Fig. 10. This may generally be determined by constructing a figure as above. It will be evident from either of these figures, that PXZ will be the sum of PXY and ZXY, or Arc 4 = Arc 2 + Arc 3.

X.—*To explain the Rule for finding the Latitude by Star Double Altitude.*

Fig. 12.



Let X be the place of the star which has the greater bearing, Y that which has the less, ♈ the first point of Aries. Draw the circles of altitude ZX*x*, ZY*y*, and the circles of declination PX*a*, PY*b*.

Then ♈*a* is the R.A. of X, and ♈*b* the R.A. of Y, and ♈*a* − ♈*b* = *ab* or YPX, the polar angle.*

* If YPX exceed 12 hours, it must be subtracted from 24 hours, and the remainder will be the polar angle.

Now, in triangle YPX we have PX, PY, and the included angle YPX, to find XY, or Arc 1.

Then with Arc 1 and the other two sides PX,PY, we can find PXY; this is Arc 2.

Again, in triangle ZXY, we have XY, or Arc 1, and the two zenith distances, ZX, ZY, to find ZXY or Arc 3.

Then PXY − ZXY = PXZ, or Arc 4.

Lastly, in triangle PXZ, with PX, ZX, and the included angle PXZ, we find PZ, the colatitude.

Whence 90° − PZ = the latitude.

THE CORRECTION FOR RUN IN THE DOUBLE ALTITUDE.

Fig. 13.

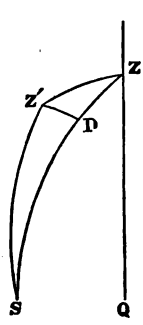
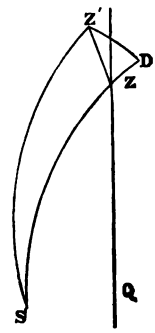


Fig. 14.



Let Z be the zenith of the place where the first observation was taken, Z′ that where the second was taken. Join ZZ′ by the arc of a great circle; draw the circles of altitude ZS, Z′S, and with S as pole, describe the arc of a parallel Z′D. Then ZZ′ will represent the ship's run, QZS the bearing of the sun at the first observation, QZZ′ the ship's course, and ZD the correction for run. Now it is evident from Fig. 13, where the angle between the ship's course and the bearing of the sun is less than 8 points, that the zenith distance at the second place is less than what it would have been at the first by the arc ZD; and from Fig. 14, where the corresponding

angle is greater than 8 points, that the zenith distance at the second place is greater than what it would have been at the first by the arc ZD. Now, as the triangle Z′ZD is very small, we may consider it as a plane triangle, right-angled at D, in which ZD = ZZ′ . Cos. Z′ZD (from Fig. 13), but in Fig. 14 Z′ZD is found by subtracting the angle between the sun's bearing and ship's course from 16 points. Hence the Rule :—

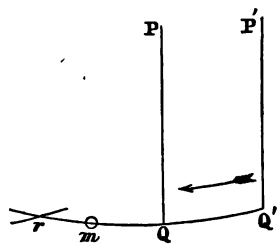
When the angle between the sun's bearing at the first observation and the ship's course is less than 8 points, add the correction to the first observed altitude. When the angle is greater than 8 points, subtract it from 16 points, and find the correction with the remaining angle, which subtract from the first observed altitude.

Cor. When Z′ZD = 90°, or 8 points ZZ′ . Cos. 90° = 0,
∴ the corr. for run is 0.

When Z′ZD = 180°, or 16 points, ZZ′ . Cos. 180° = − ZZ′,
∴ the corr. for run is − ZZ′.

XI.—*To explain and prove the Rule for finding what Stars will pass the Meridian between two given times.*

Fig. 15.



Let Υ*m*QQ′ be the equator, *m* the mean sun, and let *m*Q = the first of the given times, *m*Q′ the second. Draw the meridians PQ, PQ′ ; then

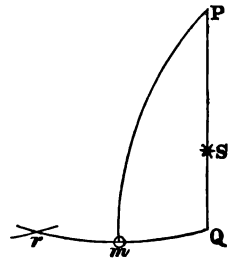
Υ*m* + *m*Q = ΥQ = Mer. R.A. corresponding to the first time, and,

Υ*m* + *m*Q′ = ΥQ′ = Mer. R.A. „ second „

Now, as the stars revolve from east to west in the direction of the arrow, that star whose R.A. is next *greater* than ΥQ will be the first to pass, and that whose R.A. is next *less* than ΥQ′ will be the last ; and all those whose R.A.s are between ΥQ and ΥQ′ will pass the meridian between the two given times.

XII.—*To explain the Rule for finding at what time any given Star will pass the Meridian of a place.*

Fig. 16.



Let γQ be the equator, PQ the meridian, m the mean sun, and let S be a star on the meridian, then will mQ be the time at the place when the star passes the meridian.

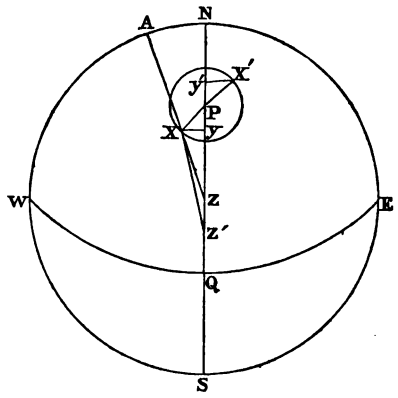
But $mQ = \gamma Q - \gamma m$,

or, Time of mer. pass = star's R.A. − R.A. mean sun,

which is the Rule.

XIII.—*To explain and prove the Rule for finding the Latitude by the Pole Star, showing how to construct a Table of Corrections.*

Fig. 17.



I. Let Z be the zenith of the observer, X the star, supposed off the meridian, draw the circle of altitude ZXA, and the circle of declination PX, and with Z as a pole, describe the arc of a parallel XY.

It is required to reduce the altitude of the star at X to what it would have been if at P, and thence to determine PN, the latitude.

Now AX, the observed altitude, is greater than PN by the part PY (nearly); PY is therefore the correction, and

PY = PZ − ZX, or, if Z′ be the zenith of a place a few degrees from Z, PY = PZ′ − Z′X, nearly. Therefore the correction calculated for latitude ZQ, will answer also for latitude Z′Q. In the triangle ZPX, we have PX, the star's N.P.D., ZPX, the star's H.A., and PZ the colat., to find XZ; this subtracted from PZ, gives PY. Let PZ = $\overline{90 - l}$, PX = $\overline{90 - d}$, and ZPX = h,

$$\text{Then, Cos. } h = \frac{\text{Cos. } z - \text{Cos. } \overline{90 - l} \,.\, \text{Cos. } \overline{90 - d}}{\text{Sin. } \overline{90 - l} \,.\, \text{Sin. } \overline{90 - d}} \qquad (a)$$

$$= \frac{\text{Cos } z - \text{Sin. } l \,.\, \text{Sin. } d}{\text{Cos. } l \,.\, \text{Cos. } d.} \qquad \text{(i and ii)}$$

Subtracting both sides from 1,

$$1 - \text{Cos. } h = 1 - \frac{\text{Cos. } z - \text{Sin, } l \,.\, \text{Sin. } d}{\text{Cos. } l \,.\, \text{Cos. } d}$$

$$\text{or Vers. } h = \frac{\text{Cos. } l \,.\, \text{Cos. } d + \text{Sin. } l \,.\, \text{Sin. } d - \text{Cos. } z}{\text{Cos. } d \,.\, \text{Cos. } d} \qquad (19)$$

$$= \frac{\text{Cos. } (l - d) - \text{Cos. } z}{\text{Cos. } l \,.\, \text{Cos. } d} \qquad (15)$$

$$\therefore \text{Cos. } l \,.\, \text{Cos. } d \,.\, \text{Vers. } h = \text{Cos. } (l - d) - \text{Cos. } z$$

$$\therefore \text{Cos. } z. = \text{Cos. } (l - d) - \text{Cos. } l \,.\, \text{Cos. } d \,.\, \text{Vers. } h.$$

Subtracting each side of the equation from 1,

$$1 - \text{Cos. } z = 1 - \text{Cos. } (l - d) + 2 \text{ Cos. } l \,.\, \text{Cos. } d \,.\, \text{Hav. } h$$

$$\text{or, Vers. } z = \text{Vers. } (l - d) + 2 \text{ Cos. } l \,.\, \text{Cos. } d \,.\, \text{Hav. } h.$$

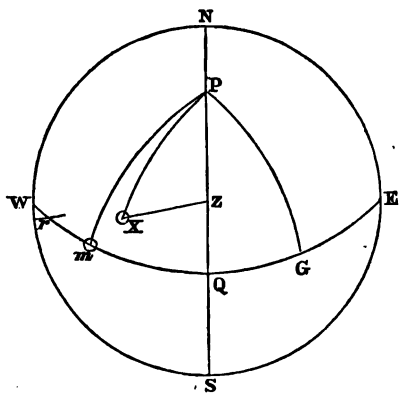
This determines ZX. Then PZ − ZX = PY, the correction for latitude ZQ or Z′Q and the intermediate latitudes. In this manner, ZP being taken at intervals of 5°, and ZPX at intervals of 10 minutes of time, the values of PY are calculated, and the results being tabulated, form the Table of Corrections for the Pole Star.

II. If the star were in the position X′, then the correction PY′ would have to be applied with a contrary sign, and it is evident that it is subtractive while the star's hour angle, east or west, is less than about 6 hours, and additive for the rest of the day.

Hence the lat. = alt. $\pm$ corr.

XIV.—*To explain and prove the Rule for finding the Longitude by Sun-Chronometer.*

Fig. 18.



Let NWSE represent the horizon, NZS the meridian, WQE the equator, P the Pole, Z the zenith of the observer, X and *m* the places of the apparent and mean sun.

Draw the circles of declination PX, P*m*, and the circle of altitude ZX. Then, in the triangle ZPX, we have given ZP, the colatitude, PX the polar distance, and ZX the zenith distance, to find ZPX, the apparent time.

First.—Let the latitude and declination be of the same name, and let PX = $\overline{90-d}$, PZ = $\overline{90-l}$, ZX = z, and ZPX = h.

$$\text{Then, } \text{Cos.}\, h = \frac{\text{Cos}\, z - \text{Cos.}\, \overline{90-d}\,.\,\text{Cos.}\, \overline{90-l}}{\text{Sin.}\, \overline{90-d}\,.\,\text{Sin.}\, \overline{90-l}} \qquad (a)$$

$$\text{Cos.}\, h = \frac{\text{Cos.}\, z - \text{Sin.}\, d\,.\,\text{Sin.}\, l}{\text{Cos.}\, d\,.\,\text{Cos.}\, l} \qquad \text{(i and ii)}$$

$$\text{and,}\quad 1-\text{Cos. } h = 1 - \frac{\text{Cos. } z - \text{Sin. } d \,.\, \text{Sin. } l}{\text{Cos. } d \,.\, \text{Cos. } l}$$

$$\text{or,}\quad \text{Vers. } h = \frac{\text{Cos. } d \,.\, \text{Cos. } l + \text{Sin. } d \,.\, \text{Sin. } l - \text{Cos. } z}{\text{Cos. } d \,.\, \text{Cos. } l} \qquad (19)$$

$$= \frac{\text{Cos. } (d-l) - \text{Cos. } z}{\text{Cos. } d \,.\, \text{Cos. } l} = -\frac{\text{Cos. } z - \text{Cos. } (d-l)}{\text{Cos. } d \,.\, \text{Cos. } l}$$

$$\therefore 2\,\text{Hav. } h = \frac{2\,\text{Sin. } \frac{\overline{d-l}+z}{2} \,.\, \text{Sin. } \frac{\overline{d-l}-z}{2}}{\text{Cos. } d \,.\, \text{Cos. } l} \qquad (17)$$

$$\therefore \text{Hav. } h = \text{Sec. } d \,.\, \text{Sec. } l \,.\, \sqrt{\text{Hav. } (\overline{d-l}+z)} \,.\, \sqrt{\text{Hav. } (\overline{d-l}-z)} \quad (\text{x\&20})$$

$$\text{or,}\quad \log. \text{Hav. } h - 10 = \log. \text{Sec. } d - 10 + \log. \text{Sec. } l - 10$$
$$+ \tfrac{1}{2}\{\log. \text{Hav. } (\overline{d-l}+z) - 10\} + \tfrac{1}{2}\{\log. \text{Hav. } (\overline{d-l}-z) - 10\}$$

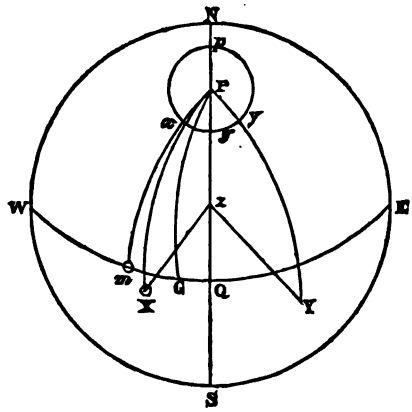
$$\therefore \log. \text{Hav. } h = \log. \text{Sec. } d + \log. \text{Sec. } l + \tfrac{1}{2} \log. \text{Hav. } (\overline{d-l}+z)$$
$$+ \tfrac{1}{2} \log. \text{Hav. } (\overline{d-l}-z) - 20,$$

which, expressed in words, is the rule for finding the apparent time, ZPX. To this apply the equation of time, XPm, and we have QPm, or Qm, ship mean time.

Now, if PG be the meridian of Greenwich, GPm or Gm will be Greenwich mean time,

and Gm − Qm = GQ, the longitude.

Fig. 19.



Secondly.—Let the latitude and declination be of different names, in which case PX will equal $90° + d$, and let the remaining parts of the figure represent the same elements as in the first case.

Then, $$\text{Cos. } h = \frac{\text{Cos. } z - \text{Cos. } \overline{90 + d} \,.\, \text{Cos. } \overline{90 - l}}{\text{Sin. } \overline{90 + d} \,.\, \text{Sin. } \overline{90 - l}} \qquad (a)$$

$$= \frac{\text{Cos. } z + \text{Sin. } d \,.\, \text{Sin. } l}{\text{Cos. } d \,.\, \text{Cos. } l} \qquad \text{(i and ii)}$$

$$\therefore \; 1 - \text{Cos. } h = 1 - \frac{\text{Cos. } z + \text{Sin. } d \,.\, \text{Sin. } l}{\text{Cos. } d \,.\, \text{Cos. } l}$$

$$\text{or, Vers. } h = \frac{\text{Cos. } d \,.\, \text{Cos. } l - \text{Sin. } d \,.\, \text{Sin. } l - \text{Cos. } z}{\text{Cos. } d \,.\, \text{Cos. } l} \qquad (19)$$

$$\therefore \; 2 \text{ Hav. } h = \frac{\text{Cos. } (d + l) - \text{Cos. } z}{\text{Cos. } d \,.\, \text{Cos. } l} \qquad (14)$$

$$\therefore \; 2 \text{ Hav. } h = \frac{2 \text{ Sin. } \frac{\overline{d + l} + z}{2} \,.\, \text{Sin. } \frac{\overline{d + l} - z}{2}}{\text{Cos. } d \,.\, \text{Cos. } l}$$

$\therefore$ Hav. h = Sec. d . Sec. l . $\sqrt{\text{Hav. } (\overline{d+l}+z)}$ $\sqrt{\text{Hav. } (\overline{d+l}-z)}$. .

as in the preceding case.

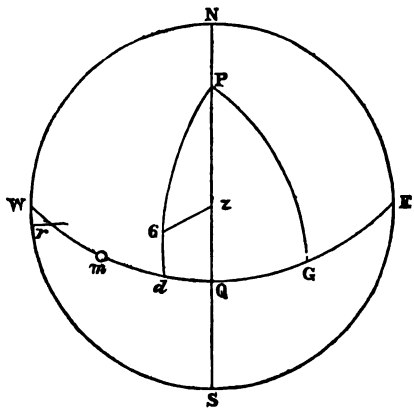
Note 1.—If PG′ be the meridian of Greenwich, G′*m* will be Greenwich mean time.

$$\therefore\ Qm - G'm = QG' = \text{the longitude (East).}$$

Note 2.—If the sun were east of meridian, the apparent time found as above would have to be subtracted from 24 hours, since *g x p y*, and *not* QPY is the sun's hour angle. This result is obtained by looking out the time at the *bottom* of the table of Haversines.

XV.—*To explain and prove the Rule for finding the Longitude by Star-Chronometer.*

Fig. 20.



Let NWSE represent the horizon, NZS the meridian, &c., and let σ be the place of the star, *m* the mean sun, Υ the first point of Aries. Draw Pσ and Zσ. Then in the triangle ZPσ we have the three sides given to find ZPσ or *d*Q the star's hour angle, as in the Sun-Chronometer.

Then, $dQ + \Upsilon d - \Upsilon m = Qm$,

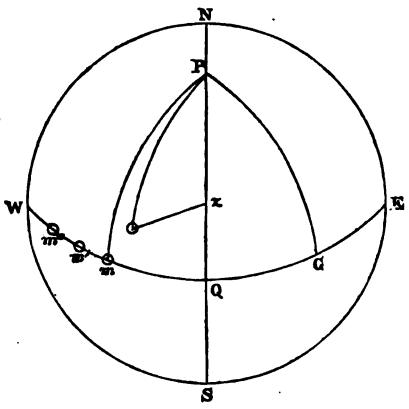
or, star's H.A. + star's R.A. − R.A.M.S. = S.M.T.

And if PG be the meridian of Greenwich, G*m* will be Greenwich mean time,

and $Gm - Qm = GQ$, the longitude.

XVI.—*To explain and prove the Rule for finding the Error of a Chronometer by a single Altitude of the Sun.*

Fig. 21.



Let NWSE be the horizon, and let the other parts of the figure represent the same elements as before, NZS being the meridian of the place of observation whose longitude GQ is accurately known.

Then, in the triangle PZX, we have given the three sides, to find the angle ZPX, as in the Chronometer Rule. This is the true time at the place where the observation is taken; to this apply the longitude in time, and we obtain G*m*, the corresponding mean time at Greenwich.

Now, suppose G*m'* to be Greenwich mean time as shown by the chronometer,

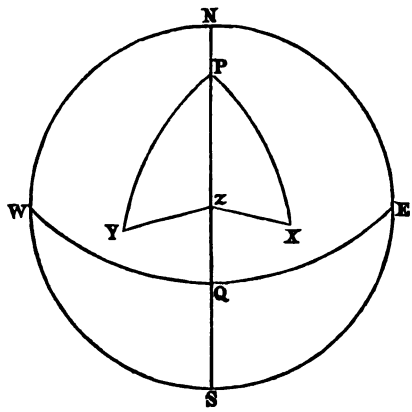
Then G*m'* — G*m* = *mm'*, the error of the chronometer.

And if *mm''* be the error obtained in the same manner after an interval of *d* days,

Then $\frac{mm'' - mm'}{d}$, or $\frac{m'm''}{d}$ = the daily rate.

XVII.—*To explain the Rule for finding the Error of a Chronometer by Equal Altitudes.*

Fig. 22.



Let t be the time by chronometer when the sun had a certain altitude *East* of meridian; t' the time when it had the same altitude *West* of meridian.

Suppose the sun to have been at X in the first case, and at Y in the second, and draw PX, PY, ZX, ZY,

Then XPY, or the elapsed time between the observations, will be equal to $t' - t$,

and XPZ, which is half the elapsed time, will be equal to $\frac{t'-t}{2}$

$\therefore\ t + \frac{t'-t}{2} = \frac{t+t'}{2}$ = the time shown *by chronometer* when the sun was on the meridian of the place of observation.

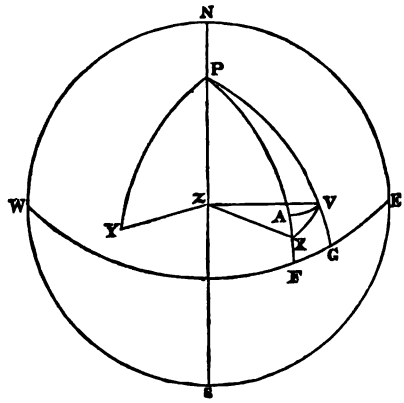
Now, we can easily find, by applying the equation of time to 0h or 24h—the time of apparent noon—the mean time when the sun was on the same meridian, which we may call T.

Then T $\sim \frac{t+t'}{2}$ will be the error of chronometer on mean time at the place of observation; but as the longitude of the place must be accurately known, we can, by applying the longitude in time, ascertain the error of the chronometer on Greenwich mean time.

THE EQUATION OF EQUAL ALTITUDES.

If the polar distances PX, PY are unequal, the angles ZPX, ZPY will also be unequal—one of them exceeding half their sum by as much as the other is less—ZPY being greater or less than ZPX, according as PY is less or greater than PX, and half the difference of these two angles is the equation of equal altitudes.

Fig. 23.



XVIII.—*To compute the Equation of Equal Altitudes.*

Let PV = PY, and ZV = ZY, or ZX; let VX be a part of a parallel of altitude, AV a part of a parallel of declination, FG the arc of the equator included between PV and PX; then AX will be the change in the polar distance in the interval measured by XPY, and $\frac{FG}{2}$, or $\frac{APV}{2}$ will be the required equation.

Now, $\frac{AV}{FG}$ = Cos. VG = Sin. PV. $\therefore$ FG = $\frac{AV}{\text{Sin. PV}}$. *(a)*

Also the right angle PVA = the right angle ZVX, and if the common angle ZVA be taken from each, the remainders PVZ, and AVX will be equal, and as AVX is very small, we may consider it as a plane triangle, right-angled at A.

$\therefore$ AV = AX . Cot. AVX = AX Cot. PVZ,

$\therefore$ FG = $\frac{AV}{\text{Sin. PV}}$ = $\frac{AX \text{ Cot. PVZ}}{\text{Sin. PV}}$ from (*a*),

or, if AX be expressed in seconds of arc, FG in seconds of time will be

$$\frac{\frac{1}{15}\text{AX} \,.\, \text{Cot. PVZ}}{\text{Sin. PV}} = \frac{\text{AX} \,.\, \text{Cot. PVZ}}{15 \,.\, \text{Sin. PV}}$$

$$\therefore \frac{\text{FG}}{2} = \frac{\text{AX Cot. PVZ}}{30 \,.\, \text{Sin. PV}}. \qquad (1)$$

But if d be the change of declination in 24 hours, and T the elapsed time between the observations,

$$\text{AX} : d :: \text{T} : 24$$

$$\text{or,} \quad \text{AX} = \frac{d \,.\, \text{T}}{24}$$

$$\text{whence} \quad \frac{\text{FG}}{2} = \frac{d\,\text{T Cot. PVZ}}{720 \,.\, \text{Sin. PV}}. \qquad \text{(from 1)}$$

Now, in triangle ZPV, let $\text{ZP} = l'$, $\text{PV} = d'$, and $\text{ZV} = z$.

$$\text{Then,} \quad \text{Cos. P} = \frac{\text{Cos.}\, z - \text{Cos.}\, d' \,.\, \text{Cos.}\, l'}{\text{Sin.}\, d' \,.\, \text{Sin.}\, l'}$$

$$\text{or,} \quad \text{Cos. P} \,.\, \text{Sin.}\, l' = \frac{\text{Cos.}\, z - \text{Cos.}\, d' \,.\, \text{Cos.}\, l'}{\text{Sin.}\, d'}$$

$$\therefore \text{Cos. P} \,.\, \text{Sin.}\, l' \,.\, \text{Cos.}\, d' = \frac{\text{Cos.}\, z \,.\, \text{Cos.}\, d' - \text{Cos.}^2 d' \,\text{Cos.}\, l'}{\text{Sin.}\, d'}$$

$$= \frac{\text{Cos.}\, z \,.\, \text{Cos.}\, d' - \text{Cos.}\, l' + \text{Sin.}^2 d' \,.\, \text{Cos.}\, l'}{\text{Sin.}\, d'} \qquad (2)$$

$$\text{Also,} \quad \text{Cos. V} = \frac{\text{Cos.}\, l' - \text{Cos.}\, z \,.\, \text{Cos.}\, d'}{\text{Sin.}\, z \,.\, \text{Sin.}\, d'}$$

$$\therefore \text{Cos. V} \,.\, \text{Sin.}\, z = \frac{\text{Cos.}\, l' - \text{Cos.}\, z \,.\, \text{Cos.}\, d'}{\text{Sin.}\, d'} \qquad (3)$$

Adding (2) and (3) together,

$$\text{Cos. P . Sin. } l\text{. Cos. } d'' + \text{Cos. V . Sin. } z = \frac{\text{Sin. } ^2d''\text{. Cos. } l'}{\text{Sin. } d''}$$
$$= \text{Sin. } d''\text{. Cos. } l' \qquad (4)$$

But, Sin. z : Sin. l' : : Sin. P : Sin. V ; or Sin. $z = \frac{\text{Sin. } l'\text{. Sin. P}}{\text{Sin. V}}$

∴ Substituting for Sin. z in (4) and reducing,

$$\text{Cos. P . Sin. } l' \text{ Cos. } d'' + \frac{\text{Cos. V . Sin. } l' \text{ Sin. P}}{\text{Sin. V}} = \text{Sin. } d'' \text{ Cos. } l'$$

$$\text{Whence} \quad \text{Cot. V} = \frac{\text{Sin. } d'' \text{ Cos. } l' - \text{Cos. P . Sin. } l'\text{. Cos. } d''}{\text{Sin. } l' \text{ Sin. P}}$$

$$= \frac{\text{Sin. } d'' \text{ Cot. } l' - \text{Cos. P . Cos. } d''}{\text{Sin. P}} \qquad (5)$$

$$\therefore \frac{\text{FG}}{2} = \frac{d\text{T}}{720\text{. Sin. } d''}\text{. Cot. V}$$

$$= \frac{d\text{T}}{720\text{. Sin. } d''}\text{.} \frac{\text{Sin. } d''\text{. Cot. } l' - \text{Cos. P . Cos. } d''}{\text{Sin. P}} \qquad \text{from (5)}$$

$$= \frac{d\text{T Cot. } l'}{720\text{. Sin. P}} - \frac{d\text{. T . Cot. } d''}{720\text{. Tan. P}}$$

$$\frac{\text{FG}}{2} = \frac{d\text{. T . Tan. lat.}}{720\text{. Sin. P}} - \frac{d\text{T . Tan. dec.}}{720\text{. Tan. P}}$$

In the table, log. A is log. $\frac{\text{T}}{720\text{. Sin. P}}$, or log. $\frac{1}{720}\text{T}^h\text{. Cosec. } \frac{\text{T}^h}{2}$

and log. B is log. $\frac{\text{T}}{720\text{. Tan. P}}$, or log. $\frac{1}{720}\text{T}^h\text{. Cot. } \frac{\text{T}^h}{2}$,

adapted to proportional logarithms.

In the case which we have supposed, where the polar distance is decreasing, the angle ZPX, which is to be added to the first chronometer time, is less than $\frac{1}{2}$XPY, by $\frac{FG}{2}$, or $\frac{FG}{2}$ is subtractive.

$$\therefore -\frac{FG}{2} = -\underset{(A)}{\frac{d.T.Cot.\ell'}{720.Sin.P}} + \underset{(B)}{\frac{d.T.Cot.d'}{720.Tan.P}}.$$

Wherefore the part under A will be — when the polar distance is decreasing; and that under B + or — according as d' is less or greater than 90°, or as the latitude and declination are of the same, or different names.*

But when the polar distance is increasing $\frac{FG}{2}$ is additive.

$$\therefore +\frac{FG}{2} = +\underset{(A)}{\frac{d.T.Cot.\ell'}{720.Sin.P}} - \underset{(B)}{\frac{d.T.Cot.d'}{720.Tan.P}}$$

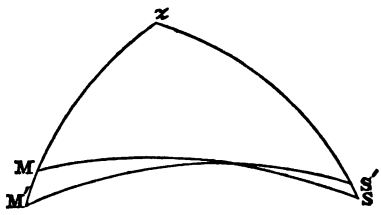
Or the part under A is + when the polar distance is increasing, and that under B is — or + according as the declination and latitude are of the same or different names.*

Hence the Rule.

* In this investigation we have supposed P not to exceed 12 hours, as in practice this will seldom or never be the case.

XIX.—*To explain the method of Clearing the Lunar Distance by the application of the first two Rules of Spherical Trigonometry.*

Fig. 24.



Let Z be the observer's zenith, M′ the apparent place of the moon, then as the moon is depressed by parallax more than it is raised by refraction, its true place will be higher, as at M. The sun, on the contrary, being raised by refraction more than it is depressed by parallax, will be below its apparent place. Let S′ be the apparent place of the sun, and S its true place,

Then M′S′ will be the apparent distance between the sun and moon,

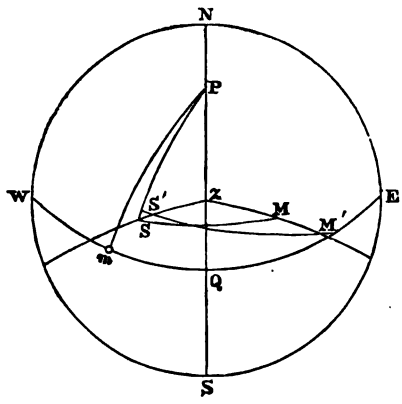
and MS the true distance.

To determine MS, we shall have, in the triangle ZM′S′, ZM′ the apparent zenith distance of the moon, ZS′ that of the sun, and M′S′ the apparent distance, to find the angle Z, by Rule I. (Three sides given, to find an angle.)

Then in the triangle ZMS we shall have ZM, the true zenith distance of the moon, ZS that of the sun, and angle Z, already found, to find MS the true distance, by Rule II. (Two sides and the included angle given, to find the third side.)

XX.—*To explain the principle of the Method of Finding the Longitude by Lunar Observation.*

Fig. 25.



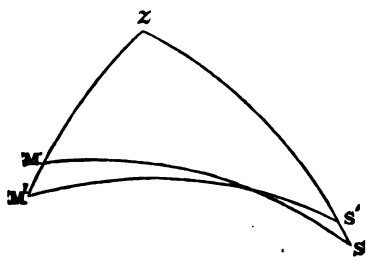
Let NWSE represent the horizon, NZS the meridian, WQE the equator, and let M, M′ be the true and apparent places of the moon, S,S′ those of the sun. Then we can, as before explained, find from the triangles MZS, M′ZS′, MS the true distance, and from the Table of Lunar Distances, given in the Nautical Almanac, we can, by interpolation, ascertain the exact time at Greenwich corresponding to this distance.

Again, in the triangle ZPS we have the three sides given, and can therefore determine ZPS, the apparent time at ship, and thence ZP*m* the S.M.T., as in the chronometer.

Whence S.M.T. ∞ G.M.T. = longitude in time.

XXI.—*To investigate the Method of Clearing the Lunar Distance by the Natural Versines.*

Fig. 26.



Let $ZS' = \overline{90° - a}$,

$ZM' = \overline{90° - a'}$ and $M'S = D$.

Also, let $ZS = z$, $ZM = z'$,

and $MS = x$.

Then, in $\triangle$ ZMS,

$$\text{Cos. } Z = \frac{\text{Cos. } x - \text{Cos. } z \,.\, \text{Cos. } z'}{\text{Sin. } z \,.\, \text{Sin. } z'} \qquad (a)$$

And, in Δ ZM'S',

$$\text{Cos. Z} = \frac{\text{Cos. D} - \text{Cos.}\,\overline{90 - a}\,.\,\text{Cos.}\,\overline{90 - a'}}{\text{Sin.}\,\overline{90 - a}\,.\,\text{Sin.}\,\overline{90 - a'}} \qquad (a)$$

$$= \frac{\text{Cos. D} - \text{Sin.}\,a\,.\,\text{Sin.}\,a'}{\text{Cos.}\,a\,.\,\text{Cos.}\,a'}$$

$$\therefore \frac{\text{Cos.}\,x - \text{Cos.}\,z\,.\,\text{Cos.}\,z'}{\text{Sin.}\,z\,.\,\text{Sin.}\,z'} = \frac{\text{Cos. D} - \text{Sin.}\,a\,.\,\text{Sin.}\,a'}{\text{Cos.}\,a\,.\,\text{Cos.}\,a'}$$

Adding, 1 to each side,

$$\frac{\text{Cos.}\,x - \text{Cos.}\,z\,.\,\text{Cos.}\,z'}{\text{Sin.}\,z\,.\,\text{Sin.}\,z'} + 1 = \frac{\text{Cos. D} - \text{Sin.}\,a\,.\,\text{Sin.}\,a'}{\text{Cos.}\,a\,.\,\text{Cos.}\,a'} + 1\,;$$

$$\text{or,}\ \frac{\text{Cos.}\,x - \text{Cos.}\,z\,.\,\text{Cos.}\,z' + \text{Sin.}\,z\,.\,\text{Sin.}\,z'}{\text{Sin.}\,z\,.\,\text{Sin.}\,z'}$$

$$= \frac{\text{Cos. D} - \text{Sin.}\,a\,.\,\text{Sin.}\,a' + \text{Cos.}\,a\,.\,\text{Cos.}\,a'}{\text{Cos.}\,a\,.\,\text{Cos.}\,a'}$$

$$\therefore \frac{\text{Cos.}\,x - (\text{Cos.}\,z\,.\,\text{Cos.}\,z' - \text{Sin.}\,z\,.\,\text{Sin.}\,z')}{\text{Sin.}\,z\,.\,\text{Sin.}\,z'}$$

$$= \frac{\text{Cos. D} + (\text{Cos.}\,a\,.\,\text{Cos.}\,a' - \text{Sin.}\,a\,.\,\text{Sin.}\,a')}{\text{Cos.}\,a\,.\,\text{Cos.}\,a'}\ ;$$

$$\text{or,}\ \frac{\text{Cos.}\,x - \text{Cos.}\,(z + z')}{\text{Sin.}\,z\,.\,\text{Sin.}\,z'} = \frac{\text{Cos. D} + \text{Cos.}\,(a + a')}{\text{Cos.}\,a\,.\,\text{Cos.}\,a'} \qquad (14)$$

$$\therefore \text{Cos.}\,x - \text{Cos.}\,(z + z') = \{\text{Cos. D} + \text{Cos.}\,(a + a')\}\,.\,\frac{\text{Sin.}\,z\,.\,\text{Sin.}\,z'}{\text{Cos.}\,a\,.\,\text{Cos.}\,a'}.$$

Now, let $\dfrac{\text{Sin.}\,z\,.\,\text{Sin.}\,z'}{\text{Cos.}\,a\,.\,\text{Cos.}\,a'} = 2\,\text{Cos. A.}$

Then, making this substitution, we shall have

$$\text{Cos.}\,x - \text{Cos.}\,(z + z') = \{\text{Cos. D} + \text{Cos.}\,(a + a')\}\ 2\,\text{Cos. A}$$

$$\therefore\ \text{Cos.}\,x - \text{Cos.}\,(z + z') = 2\,\text{Cos. D . Cos. A}$$

$$+ 2\,\text{Cos.}\,(a + a')\,.\,\text{Cos. A}$$

$$\text{or, Cos.}\,x - \text{Cos.}\,(z + z') = \text{Cos.}\,(\text{D} + \text{A}) + \text{Cos.}\,(\text{D} - \text{A})$$

$$+ \text{Cos.}\,(a + a' + \text{A}) + \text{Cos.}\,(a + a' - \text{A}) \qquad (18)$$

$$\therefore\ \text{Cos.}\,x = \text{Cos.}\,(z + z') + \text{Cos.}\,(\text{D} + \text{A}) + \text{Cos.}\,(\text{D} - \text{A})$$

$$+ \text{Cos.}\ (a + a' + \text{A}) + \text{Cos.}\,(a + a' - \text{A}).$$

Subtracting each term from 1, we have

$$1 - \text{Cos.}\,x = 1 - \text{Cos.}\,(z + z') + 1 - \text{Cos.}\,(\text{D} + \text{A}) + 1 - \text{Cos.}\,(\text{D} - \text{A})$$

$$+ 1 - \text{Cos.}\,(a + a' + \text{A}) + 1 - \text{Cos.}\,(a + a' - \text{A}) - 4$$

$$\text{or, Vers.}\,x = \text{Vers.}\,(z + z') + \text{Vers.}\,(\text{D} + \text{A}) + \text{Vers.}\,(\text{D} - \text{A})$$

$$+ \text{Vers.}\,(a + a' + \text{A}) + \text{Vers.}\,(a + a' - \text{A}) - 4$$

$$\text{or, Tab. Vers.}\,x = \text{Tab. Vers.}\,(z + z')$$

$$+ \text{Tab. Vers.}\,(\text{D} + \text{A}) + \&\text{c.} \qquad - 4000000$$

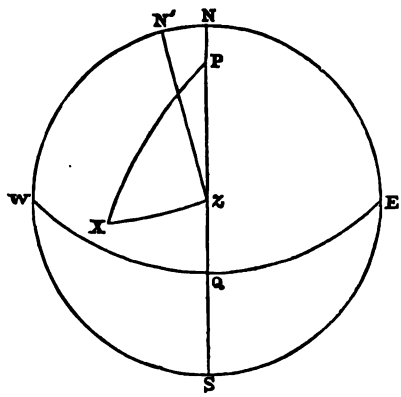
(Each term being multiplied by 1000000).

This, expressed in words, is the Rule.

Many other methods have been proposed for clearing the Lunar Distance, but as few can equal this for elegance, simplicity, and accuracy combined, it has not been thought expedient to enter into them.

XXII.—*To explain and prove the Rule for finding the Variation by Azimuth.*

Fig. 27.



Let NWSE represent the horizon, and let the other parts of the figure represent the same elements as before, then PZX, the sun's azimuth or true bearing, is found with the three sides of the spherical triangle PZX.

Let PZ $= \overline{90° - l}$, ZX $= \overline{90° - a}$, PX $= p$, and angle PZX $= z$

$$\text{Then Cos.}\, z = \frac{\text{Cos.}\, p - \text{Cos.}\, \overline{90 - a}\, .\, \text{Cos.}\, \overline{90 - l}}{\text{Sin.}\, \overline{90 - a}\, .\, \text{Sin.}\, \overline{90 - l}} \qquad (a)$$

$$\text{or,}\quad \text{Cos.}\, z = \frac{\text{Cos.}\, p - \text{Sin.}\, a\, .\, \text{Sin.}\, l}{\text{Cos.}\, a\, .\, \text{Cos.}\, l}$$

$$\therefore\ 1 - \text{Cos.}\, z = 1 - \frac{\text{Cos.}\, p - \text{Sin.}\, a\, .\, \text{Sin.}\, l}{\text{Cos.}\, a\, .\, \text{Cos.}\, l}$$

$$\therefore\ \text{Vers.}\, z = \frac{\text{Cos.}\, a\, .\, \text{Cos.}\, l + \text{Sin.}\, a\, .\, \text{Sin.}\, l - \text{Cos.}\, p}{\text{Cos.}\, a\, .\, \text{Cos.}\, l}$$

$$\text{or, 2 Hav.}\, z = \frac{\text{Cos.}\, (a - l) - \text{Cos.}\, p}{\text{Cos.}\, a\, .\, \text{Cos.}\, l} = \frac{2\ \text{Sin.}\, \frac{\overline{a - l + p}}{2}\ \text{Sin.}\, \frac{\overline{a - l - p}}{2}}{\text{Cos.}\, a\, .\, \text{Cos.}\, l}$$

$$\therefore\ \text{Hav.}\, z = \text{Sec.}\, a\, .\, \text{Sec.}\, l \sqrt{\text{Hav.}\, \overline{(a - l + p)}}\, .\, \sqrt{\text{Hav.}\, \overline{(a - l - p)}}$$

Whence log. Hav. z = log. Sec. a + log. Sec. l

+ ½ log. Hav. $(\overline{a - l} + p)$ + ½ log. Hav. $(\overline{a - l} - p)$—20

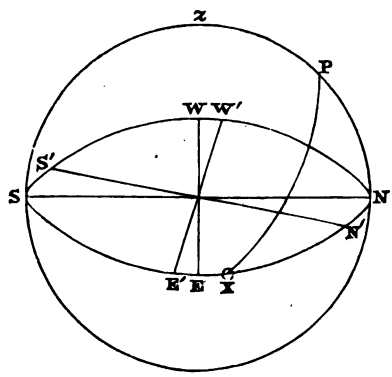
which, in words, is the rule for finding the true azimuth.

Now, suppose ZN′ to be a part of the magnetic meridian, then will N′ZX be the magnetic azimuth.

$\therefore$ NZX − N′ZX = NZN′ or NN′, the variation.

XXIII.—*To explain and prove the Rule for finding the Variation by Amplitude.*

Fig. 28.



Let SZN be the meridian, NESW the horizon, and let N′E′S′W′ be the cardinal points as shown by the compass. Suppose the sun to be at X, at rising, then EX will be its true amplitude, E′X its magnetic amplitude. Draw the circle of declination PX, then in the right-angled triangle PNX,

We shall have Cos. PX = Cos. PN . Cos. XN,

or, Sin. dec. = Cos. lat. . Sin. amp.

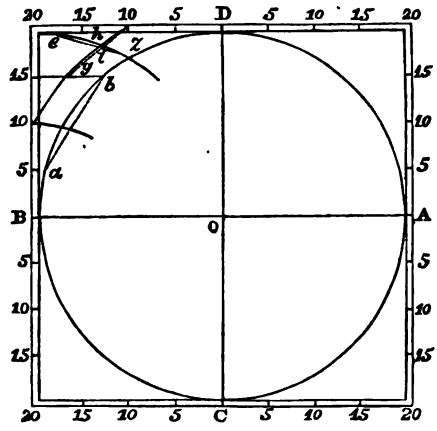
Whence Sin. amp. = Sin. dec. . Sec. lat.

From this formula we obtain the true amplitude, EX.

Then EX − E′X = EE′ or NN′, the variation.

XXIV.—*To explain Sumner's Method of finding the Latitude and Longitude.*

Fig. 29.



By this method two sets of chronometer sights are taken at an interval of not less than 1½ or 2 hours—the second set being corrected for run, as in the double altitude. Each set is then worked with two assumed latitudes (between which we certainly know our true latitude to lie). Then if the position of the ship, for the four different longitudes thus found, be laid down on a chart, the point of intersection of the line joining the first and second with that joining the third and fourth will be the true position of the ship at the second observation.

I. To make this understood, suppose, for simplicity's sake, the sun is vertical at O, the point of intersection of the meridian of Greenwich with the equator, and with O as a centre and a radius equal to 20°, say, describe a circle; then if at A,B,C,D the sun's altitude were taken at the same instant, each observer would find the zenith distance to be 20°, ABCD is therefore a circle of *equal zenith distance*, and if his chronometer be correct it will place him on this circle when he finds the zenith distance to be 20°, because it shows the sun to be at O, when it ought to be. Let us now suppose that the ship's latitude is somewhere between 5° N. and 15° N., and the time A.M.; then working out his longitude with 5° N., his chronometer will place him at *a*, the point where the parallel of 5° intersects the circle, and working it with 15° N. it will place him at *b*, the point where the parallel of 15° cuts the circle, he must therefore be somewhere on the arc between *a* and *b*.

Again, suppose the observer to take the altitude a second time, after an interval of 1h. 20m. say, by which time the sun will have reached the meridian of B, and let the zenith distance be 10°, suppose, and with B as a centre, and 10° as a radius, describe the arc of another circle of equal zenith distance. Then, as the observer must be on each of these circles, he will be at their point of intersection. When the latitudes do not differ by more than one or two degrees the chord joining *a* and *b* will pass nearly through the place of the observer; *ab* is therefore a *line of position.*

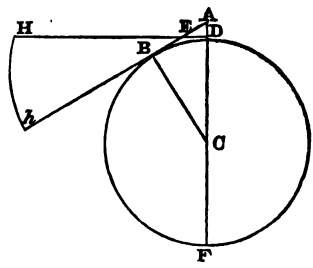
II. Suppose the G.M.T. to be 22h., the dec. to be 20° S. and Z.D. 59° N., and the assumed latitudes to be 16° 15′ N. and 17° 45′ N., and let *g* and *h* be the positions given by these data, and suppose that 4 hours afterwards, or 2h. G.M.T., the Z.D. was 40°, and the same operation being repeated with the same latitudes, the two positions would be *z* and *e*, and *i*, their point of intersection, would be the position of the ship, nearly. Its true position would be the intersection of the arcs themselves, but the error, being small, may be disregarded. In the same way we might find the place of the ship by observations of the sun and moon, or of two stars whose difference of bearing is not less than 45°, nor greater than 135°, for if so the two chords, *gh*, *ze*, would cut at too acute an angle to show the position accurately.

Cor.—The line of position *ab*, when the latitudes do not differ greatly, is the chord of a very small arc; a perpendicular bisecting this line would pass through the centre of the zenith distance circle, therefore a perpendicular to either line of position drawn on the chart would show the azimuth of the sun at the time of observation, and if the magnetic azimuth be taken with the altitude, the variation may be determined without calculation, sufficiently near to steer by.

Note.—In the preceding explanation we have spoken of the Fig. ABCD as a circle, but it is evident that, if described upon a Mercator's chart, it would be an ellipse; the principle, however, is exactly the same.

XXV.—*To investigate an Expression for Dip.*

Fig. 30.



Let Hh be a portion of the celestial concave, DBF a section of the earth, AD = a, the height of the observer's eye above it. Take C, the centre of the earth, produce AD through C to F, draw the tangents DEH, AEh, and join CB. Then Ah will be the visible horizon, and DEH the true horizon, and Hh (or HEh = AED = ACB) will be the dip.

$$\therefore \text{Tan. dip} = \text{Tan. HE}h = \text{Tan. AED} = \text{Tan. ACB} = \frac{\text{AB}}{\text{BC}}$$

$$= \frac{\sqrt{\text{AD} \cdot \text{AF}}}{\text{BC}} \text{ (Euc. III. 36)} = \frac{\sqrt{a(a+2r)}}{r} = \frac{\sqrt{a^2+2ar}}{r} = \frac{\sqrt{2ar}}{r}$$

(Neglecting a^2, which is very small compared with $2ar$, r being the earth's radius)

$$\text{Tan. dip} = \sqrt{\frac{2ar}{r^2}} = \sqrt{\frac{2a}{r}}. \qquad (1)$$

But also, Tan. dip = $\frac{\text{H}h}{\text{HE}}$, nearly, since HE$h$ is very small; and HE may be considered as the radius of the circle of which Hh is an arc; and the radius of a circle is equal to $57^{\circ}{\cdot}5957$, or $57^{\circ}5957 \times 60$ minutes of its circumference.

$$\therefore \text{Tan. dip} = \frac{\text{H}h \text{ (in minutes)},}{\text{HE (in minutes)},} = \frac{\text{dip in minutes}}{57{\cdot}5957 \times 60} \qquad (2)$$

$\therefore$ Equating (1) and (2),

$$\frac{\text{dip in minutes}}{57{\cdot}5957 \times 60} = \sqrt{\frac{2a}{r}} = \frac{\sqrt{2a}}{\sqrt{3959 \times 1760 \times 3}}$$

$$\therefore \text{The dip} = \frac{57^{\circ}{\cdot}2957 \times 60\sqrt{2}}{\sqrt{3959 \times 1760 \times 3}} \cdot \sqrt{a}$$

But the dip has been found by experiment to be diminished by refraction to the amount of $\frac{3}{40}$ of itself.

$$\therefore \text{ the true dip} = \frac{37}{40} \cdot \frac{57^{\circ}\cdot 2957 \times 60 \times \sqrt{2}}{\sqrt{3959 \times 1760 \times 3}} \cdot \sqrt{a}$$

$$= \cdot 984\sqrt{a}, \text{ nearly.}$$

Ex. To compute the dip for 20 feet.

$$\text{dip} = \cdot 984\sqrt{20}$$

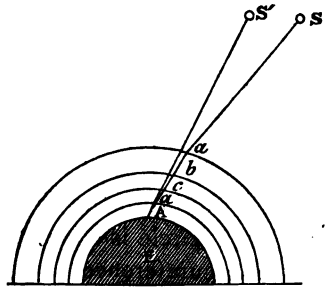
$$\log. \cdot 984 = \bar{1}\cdot 992995$$

$$\log. \sqrt{20} = \cdot 650515$$

$$\log. \text{ dip} = \cdot 643510 \qquad \therefore \text{ dip} = 4'\cdot 4 = 4'\cdot 24''$$

ON REFRACTION.

Fig. 31.

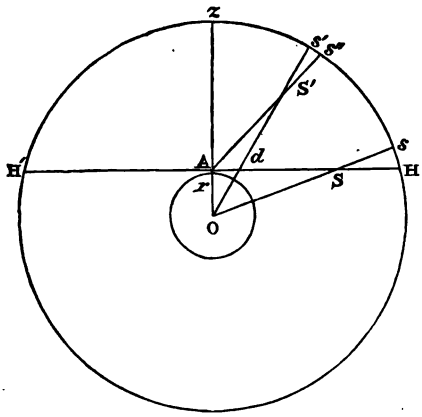


When a ray of light passes obliquely from one medium to another of greater density it is bent towards the perpendicular at the point at which it enters. This bending is called refraction. Now the strata of the atmosphere through which the rays of light pass to the earth increase in density as they approach the surface, being compressed by the weight of the superincumbent atmosphere. Therefore, if S be the sun and S*a* a ray of light proceeding from it, in its passage from S to *a* it will be a straight line, but at *a* it will be refracted, or bent down in the direction *ab*, and at *b* it will be bent still more, the density of this stratum being greater than that of the one above it, so that the path of the ray will be similar to S*abcd*. Now, if we conceive these strata to be indefinitely thin, the points *abc*... will be indefinitely near to each other, and *abcd*... will be a curve, and

the direction in which S will be seen will be in that of the tangent to this curve where it meets the observer's eye. Except near the horizon the refraction at different altitudes is found to be nearly proportional to the cotangents of the altitudes. In a mean state of the atmosphere (*i.e.*, Bar. 30 in., Ther. 50°) the refraction at the horizon is 33′.

XXVI.—*To prove that Parallax in Alt. = Hor. Par. × Cos. Alt.*

Fig. 32.



Let O be the earth's centre, A the place of the observer, Z his zenith, HAH′ the horizon, S the place of the body in the horizon, S′ above it. Join OS, OS′ and AS′; then will ASO be the horizontal parallax, and AS′O, the parallax in altitude.

Let AO = r, OS and OS′ = d, ASO = P, and AS′O = p.

Then, $\text{Sin. P} = \frac{r}{d}$; and $\frac{\text{Sin.}\,p}{\text{Sin. S′AO}} = \frac{r}{d}$

$$\therefore \frac{\text{Sin.}\,p}{\text{Sin. SA′O}} = \text{Sin. P.}$$

Or, $\text{Sin.}\,p = \text{Sin. P . Sin. S′AO} = \text{Sin. P . Sin. ZAS′}$

$= \text{Sin. P . Cos. S′AH.}$

Or, Since p and P are very small,

$$p = \text{P . Cos. S′AH}$$

i. e., Parallax in alt. = hor. parallax × Cos. alt.

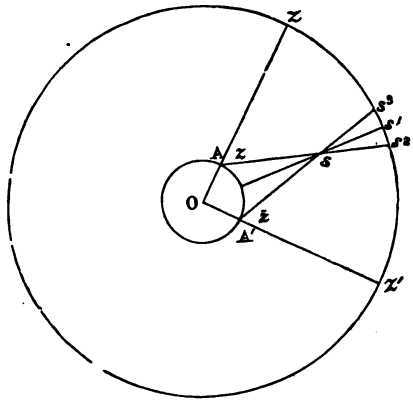
Ex. Given, moon's hor. par. 60′ and alt. 30°; compute the parallax in alt.

log. 60′ = 1·778151
log. Cos. 30° = 9·937531

log. p = 1·715682
p = 51′·96 = 51′·57″.

XXVII.—*To explain the Method of finding the Parallax of a Heavenly Body.*

Fig. 33.



Let O be the centre of the earth; Zs_1Z', the celestial concave; A,A′ the positions of two observers, supposed to be on the same meridian; Z,Z′ their zeniths; s the heavenly body, which an observer at O would refer to s_1 on the celestial concave, and others at A and A′ to s_2 and s_3—then s_1s_2 and s_1s_3 are the parallax in alt. at A and A′ respectively.

$\therefore$ s_1s_2 = P . Sin. z; and s_1s_3 = P . Sin z' (by preceding art.)

But, s_1s_2, or P Sin. z = $Zs_2 - Zs_1$ (1)

And, s_1s_3, or P Sin. z' = $Z's_3 - Z's_1$ (2)

$\therefore$ Adding (1) and (2) together,

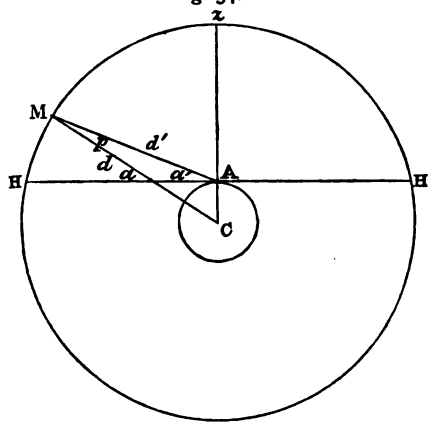
$$P\,.\,(\text{Sin. } z + \text{Sin. } z') = (Zs_2 + Z's_3) - (Zs_1 + Z's_1)$$

$$\text{Or, } P = \frac{(Zs_2 + Z's_3) - (Zs_1 + Z's_1)}{\text{Sin. } z + \text{Sin. } z'}$$

In this equation Zs_2, $Z's_3$, z and z' are determined by observation, and $Zs_1 + Z's_1$ is equal to the difference of latitude, or to the sum of the latitudes, if the places be on opposite sides of the equator. Hence P is completely determined.

XXVIII.—*To compute the Augmentation of the Moon's Semidiameter.*

Fig. 34.



Let A be the place of the observer, Z his zenith, HAH' the horizon, M the moon; let MC, its distance from the earth's centre, $= d$; AM, its distance from the observer, $= d''$; $s =$ the diameter at the distance d, s', at the distance d''; then $s' - s$ will be the augmentation.

Let $a =$ the app. alt.; $a' =$ the true alt.

$$\text{Then } \frac{d}{d'} = \frac{s'}{s}; \text{ and } \frac{d}{d''} = \frac{\text{Cos. } a}{\text{Cos. } a'}$$

$$\therefore \frac{s'}{s} = \frac{\text{Cos. } a}{\text{Cos. } a'}$$

$$\therefore \frac{s'-s}{s} = \frac{\text{Cos. } a - \text{Cos. } a'}{\text{Cos. } a'} = 2\,\text{Sin.}\tfrac{1}{2}(a'+a).\,\text{Sin.}\tfrac{1}{2}(a'-a).\,\text{Sec. } a';$$

or, $s' - s = 2s\,.\,\text{Sin.}\,\tfrac{1}{2}(a' + a)\,.\,\text{Sin.}\,\tfrac{1}{2}(a' - a)\,.\,\text{Sec.}\,a'$ (1)

Now, from the fig. $a' = a + p$

or, $a' - a = p$

$$\therefore \tfrac{1}{2}(a' - a) = \tfrac{1}{2}p. \qquad (2)$$

Subtracting each side of this equation from a', we have

$$a' - \tfrac{1}{2}(a' - a) = a - \tfrac{1}{2}p$$

$$\text{or, } \tfrac{1}{2}(a' + a) = a' - \tfrac{1}{2}p \qquad (3)$$

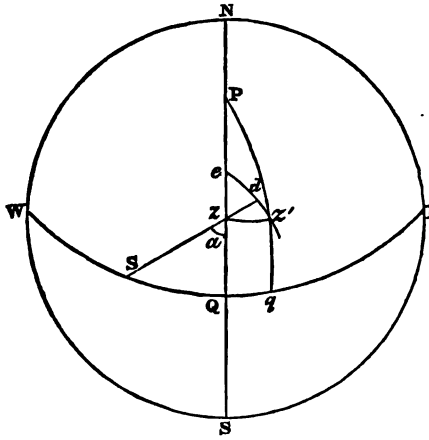
Substituting the equivalent values of $\frac{1}{2}(a' - a)$ and $\frac{1}{2}(a' + a)$ obtained from (2) and (3), in equation (1),

$$s' - s = 2s \,.\, \text{Sin.}(a' - \tfrac{1}{2}p) \,.\, \text{Sin.}\,\tfrac{1}{2}p \,.\, \text{Sec.}\, a'.$$

From which expression the table for augmenting the moon's semidiameter is calculated.

XXIX.—*To find the Error in Longitude resulting from a Small Error in the Altitude of a Heavenly Body.*

Fig. 35.



Let S be the heavenly body from whose altitude the longitude is to be found, Z the true zenith of the observer; draw the circle of altitude SZ*d*, making Z*d* equal to the error in altitude, and with S as pole describe the parallel *ed*Z' through *d*, cutting that through Z in Z', and draw the meridian PZ'*q*; then will ZZ' represent the error in departure, and *Qq* the error in longitude, corresponding to the error in altitude Z*d*. Let the azimuth SZQ = *a*, and the lat. ZQ = *l*.

Now the triangle ZZ'*d* being very small, may be considered as a plane triangle, right-angled at *d*,

$$\text{And} \quad \text{ZZ}'d = 90^\circ - d\text{ZZ}' = \text{EZ}d = a$$

$$\therefore \text{ZZ}' = \frac{\text{Z}d}{\text{Sin. ZZ}'d} = \frac{\text{Z}d}{\text{Sin.}\, a}$$

But also, $ZZ' = Qq \,.\, \text{Cos. } ZQ = Qq \,.\, \text{Cos. } l$

$$\therefore \; Qq \,.\, \text{Cos. } l = \frac{Zd}{\text{Sin. } a}$$

$$\text{or,} \quad Qq = \frac{Zd}{\text{Sin. } a \,.\, \text{Cos. } l} = Zd \,.\, \text{Cosec. } a \,.\, \text{Sec. } l$$

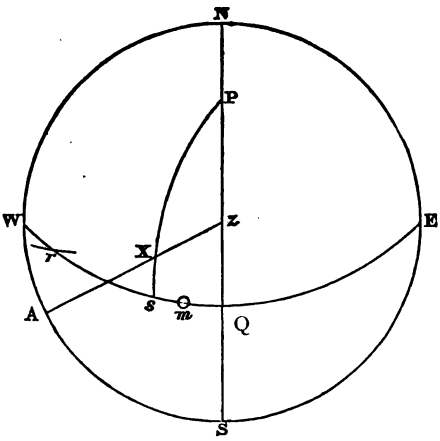
i.e., Error in long. = error in alt. × Cosec. a × Sec. lat.

∴ As regards the azimuth, the error in longitude is least when Cosec. a is least, *i.e.*, when $a = 90°$; and as regards the latitude, the error is least when Sec. l is least, *i.e.*, when $l = 0°$.

Cor. It is evident from the foregoing considerations, that when the body is *west* of meridian, an error of defect in the altitude will place the observer too far to the eastward, and that one of excess will place him too far to the westward, and that the reverse will be the case when the body is *east* of meridian.

XXX.—*To explain and prove the Rule for Calculating the Altitude of a Heavenly Body.*

Fig. 36.



Let NWSE represent the horizon, &c., as before; and let X be the place of the heavenly body whose altitude, AX, is to be found.

We must first find the hour angle of the body—if it be the sun, by taking the G.M.T. of observation by chronometer, and by applying the longitude in time, thus :—

S.M.T. = G.M.T. ± longitude in time according as the

longitude is E. or W.: to the time thus found, by applying the equation of time, we obtain the sun's hour angle, ZPX.

If the body be the moon, or a star, we have

$$sQ = \Upsilon Q - \Upsilon s = mQ + \Upsilon m - \Upsilon s$$

Or, H.A. = S.M.T. + R.A.M.S. − Star's H.A.

Let ZPX = h, PX = $\overline{90 - d}$, PZ = $\overline{90 - l}$, and ZX = z.

$$\text{Then, } \text{Cos.}\, h = \frac{\text{Cos.}\, z - \text{Cos.}\, \overline{90 - d}\,.\,\text{Cos.}\, \overline{90 - l}}{\text{Sin.}\, \overline{90 - d}\,.\,\text{Sin.}\, \overline{90 - l}} \qquad (a)$$

$$= \frac{\text{Cos.}\, z - \text{Sin.}\, d\,.\,\text{Sin.}\, l}{\text{Cos.}\, d\,.\,\text{Cos.}\, l}$$

Subtracting each side from 1,

$$1 - \text{Cos.}\, h = 1 - \frac{\text{Cos.}\, z - \text{Sin.}\, d\,.\,\text{Sin.}\, l}{\text{Cos.}\, d\,.\,\text{Cos.}\, l}$$

$$= \frac{\text{Cos.}\, d\,.\,\text{Cos.}\, l + \text{Sin.}\, d\,.\,\text{Sin.}\, l - \text{Cos.}\, z}{\text{Cos.}\, d\,.\,\text{Cos.}\, l}$$

$$\text{Or, } \text{Vers.}\, h = \frac{\text{Cos.}\, (d - l) - \text{Cos.}\, z}{\text{Cos.}\, d\,.\,\text{Cos.}\, l}$$

$$\therefore \text{Cos.}\, d\,.\,\text{Cos.}\, l\,.\,\text{Vers.}\, h = \text{Cos.}\, (d - l) - \text{Cos.}\, z$$

$$\therefore -\text{Cos.}\, z = -\text{Cos.}\, (d - l) + \text{Cos.}\, d\,.\,\text{Cos.}\, l\,.\,\text{Vers.}\, h$$

Adding 1 to each side,

$$1 - \text{Cos.}\, z = 1 - \text{Cos.}\, (d - l) + 2\,\text{Cos.}\, d\,.\,\text{Cos.}\, l\,.\,\text{Vers.}\, h$$

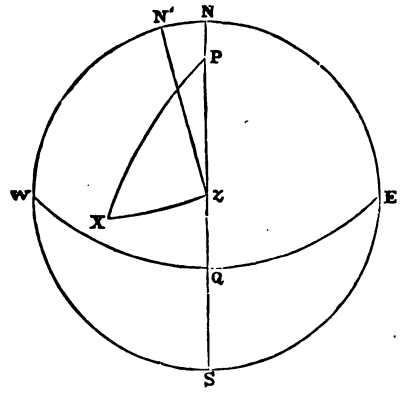
$$\therefore \text{Vers.}\, z = \text{Vers.}\, (d - l) + 2\,\text{Cos.}\, d\,.\,\text{Cos.}\, l\,.\,\text{Vers.}\, h$$

Which, in words, is the rule for finding ZX or z the zenith distance.

Whence $90^\circ - z$ = AX, the alt.

XXXI.—*To explain and prove the Rule for finding the Variation of the Compass by Time Azimuth.*

Fig. 37.



Let ZPX, the apparent time at place, $= h$; PZ, the Colat. $= l'$; and PX, the polar distance, $= p$.

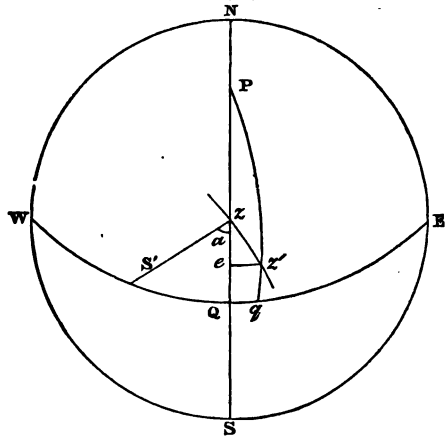
Then, $\text{Tan.}\ \frac{1}{2}(Z + X) = \text{Sin.}\ \frac{1}{2}(p - l')\ .\ \text{Cosec.}\ \frac{1}{2}(p + l')\ .\ \text{Cot.}\ \frac{h}{2}$

and $\text{Tan.}\ \frac{1}{2}(Z - X) = \text{Cos.}\ \frac{1}{2}(p - l')\ .\ \text{Sec.}\ \frac{1}{2}(p + l')\ \text{Cot.}\ \frac{h}{2}$

Combining which equations we obtain NZX, the true azimuth; and if N′ZX be the magnetic azimuth, NZX − N′ZX = NZN′ or NN′, the variation.

XXXII.—*To find the Error in Longitude resulting from a small Error in the Latitude.*

Fig. 38.



Let S′ be the heavenly body, from an altitude of which the longitude is to be found, Z the zenith of the observer, and *Ze* the error in latitude; with S′ as pole describe a parallel through Z cutting that through *e* in Z′, and draw the meridian PZ′*q*; then *eZ*′ will be the error in departure, and *Qq* the error in longitude, corresponding to *Ze*, the error in latitude.

Let the azimuth S′ZQ = *a*; and the lat. EQ = *l*.

Now, ZZ′*e* being very small, may be considered as a plane triangle, right-angled at *e*; and S′ZZ′ may also be considered as a right angle; and $ZZe = 90° - eZZ' = a$

$$\therefore\ eZ' = Ze \,.\, \text{Cot. } a\ ;\ \text{and } eZ' = Qq \,.\, \text{Cos. } l$$

$$\therefore\ Qq \,.\, \text{Cos. } l = Ze \,.\, \text{Cot. } a$$

$$\text{or,}\quad Qq = Ze \,.\, \text{Cot. } a \,.\, \text{Sec. } l$$

i.e., Error in longitude = error in lat. × Cot. *a* . × Sec. lat.

∴ As regards the azimuth, the error in long. is least when Cot. *a* is least, *i.e.*, when $a = 90°$; and as regards the lat., the error is least when Sec. *l* is least, or when $l = 0°$.

Cor. It is evident from the above considerations, that an error south in latitude will place the observer to the eastward of his true position, and an error north will place him to the westward, when the body is *west* of meridian, and the azimuth from S less than 90°, and that when the azimuth from N is less than 90° the reverse will be the case.

When the body is *east* of meridian the converse of the foregoing reasoning will be true.

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